Modeling Car-following Using System Dynamics

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Car following models describe driver behavior in a traffic stream and therefore form a key component of all microscopic traffic simulation models. A large number of following models have been proposed and are described in the literature, however, many of these models are based on unrealistic assumptions of drivers' abilities and/or the models make use of relationships that do not correspond to physical aspects of the car-following process.

In this paper we introduce a new Systems Dynamic (SD) car-following model that addresses many of the shortcomings of existing car-following models. While the proposed model was validated using field data obtained from an arterial roadway, the model structure is appropriate for all roadway types. The validation results suggest that the proposed model yields speed and spacing profiles for vehicles in "real time" that compare well with those observed empirically.

Keywords: Car-following, Driver behavior, Systems Dynamics, Microscopic traffic simulation

INTRODUCTION

The development and consideration of a wide range of new technologies including in-vehicle guidance, driver vision aids, collision warning systems, adaptive cruise control, etc. requires the evaluation of these systems in terms of their impacts on traffic flow performance and safety. These technologies are expected to modify driver behavior in a complex interactive fashion. One of the ways in which these and other systems or policies can be evaluated is through the use of simulation models. However, to simulate the effects of the proposed technology or policy accurately, the simulation model must be able to capture the changes that the proposed technology or policy has on driver behavior.

Driver behavior involves two main responses: 1) speed and 2) steering. The primary objective of most car-following models is to predict following vehicle speed and spacing profiles based on lead vehicle stimuli (speeds) for a set of route/traffic conditions and driver characteristics. These models typically consider a string of vehicles traveling in a single lane. Lane changes are normally not considered within the scope of car-following algorithms. More complex driver responses considered within more extensive microscopic traffic simulations combine car-following models with models of other driver responses (i.e. lane changes, routing, etc.) to produce a more practical topology of driver behavior in actual traffic situations.

The model introduced in this paper makes use of Systems Dynamics (SD) principles. Systems Dynamics provides the computational platform for describing and investigating the complex process that reflect driver behavior in a traffic stream. The SD platform is characterized by many non-linear relationships (both heuristic and empirical) with numerous feedback loops. As such, the proposed SD car-following model introduced in this paper relaxes many of the limiting assumptions of existing car-following models, rendering the process more relevant for microscopic traffic simulation.

This paper has two objectives: 1) develop a SD car-following model that addresses many of the shortcomings identified in existing model, and 2) compare the SD model to observed vehicle tracking data and assess its ability to predict speed and spacing profiles over time.
REVIEW OF CAR-FOLLOWING MODELS

Car following models determine the acceleration (or deceleration) rate of the following vehicle in a given time interval based on the actions of the lead vehicle(s). Once the acceleration or deceleration rate of the following vehicle is determined, equations of motion are used to compute the speed and the position of the following vehicle for any given time interval.

Over the past 50 years, many different car-following models have been proposed to describe driver behavior in a traffic stream. Driver behavior literature suggests four different types of car-following models. These four types are 1) Stimulus-response car-following models, 2) Safety-distance or behavioral car-following models, 3) Psychophysical or action point car-following models, and 4) Fuzzy logic-based car-following models.

A comprehensive review of the historical development of car-following models is available in Brackstone and McDonald (1). A more recent review by Mehmood et al. (2) identified the following four assumptions that are inherent in many existing models and tend to restrict the models' ability to explain and predict driver behavior in actual traffic situations:

1. The vast majority of car-following models assume that following vehicle drivers can accurately perceive relative speed of the lead and following vehicles, absolute speed and/or acceleration of lead vehicle at any point in time. These assumptions, particularly the assumption of being able to perceive absolute speed and acceleration, are unrealistic given the rectilinear nature of vehicles moving in a single lane, and problems of depth perception and different driver reactions with factors such as, aging, impairment, disability, etc (3).
2. Many existing car-following models assume that following vehicle drivers respond only to the lead vehicle immediately in front without observing other vehicles downstream. A number of researchers have observed that in actual traffic situations, drivers take a more extensive view of traffic conditions ahead (which may include several lead vehicles) in setting the following vehicle desired speeds and spacing (4-7).
3. Many existing car-following models, particularly the stimulus-response models, assume a mathematical expression that is empirically based but fails to explain actual behavior in a mechanistic fashion (cause-effect). Best-fit expressions fail to clarify or explain why certain relationships are specified (8). These expressions have little, if any, basis on actual behavior, and the model parameters have no obvious connection with identifiable driver and vehicle traits that explains behavior (9).
4. Many existing stimulus-response car-following models assume symmetrical driver responses to changing traffic stimuli involving lead vehicles. This is often unrealistic, as the impetus to decelerate when separation distance is smaller than desired is based on safety (i.e. collision avoidance) considerations and is likely to be more forceful than the impetus to accelerate when the separation distance is larger than required for the current speed. Furthermore, when both the lead and following vehicle are traveling at the same speed, many existing car-following models assume zero following vehicle deceleration/acceleration rates regardless of the spacing between vehicles. This assumption is clearly unrealistic (10).

PROPOSED SD CAR-FOLLOWING MODEL

The car-following situation considered in this paper assumes a string of three vehicles (two lead vehicles and one following vehicle) traveling along a single lane. It is assumed that all vehicles travel in the same lane and only adjustments in speed are permitted for all drivers involved. The speed profiles of the first and second lead vehicle are assumed to be provided externally. The proposed car-following model determines internally the acceleration/deceleration rate, speed, and spacing profiles of the following vehicle on the basis of the behaviour of the downstream lead vehicle(s).

Underlying assumptions

Figure 1 illustrates the underlying assumptions of the proposed car-following model. It is assumed that the acceleration/deceleration rate of the following vehicle driver depends on the current speed, the control
speed, and the perception-reaction time of the following vehicle driver. The control speed is defined as the maximum speed at which the following vehicle driver would travel given the spacing and rate of change in spacing with respect to the lead vehicle immediately in front (i.e. second lead vehicle). A number of researchers, for example, Brown (11), Richard et al. (12), Gordon et al. (13), and Konishi et at. (14) reported that because of human limitations, changes in speed of the lead vehicle are not detectable. Conversely, spacing between successive vehicles is a rich visual input, which is relatively easy for the following vehicle drivers to ascertain in car-following situations. Consequently, the proposed car-following model assumes that the following vehicle driver can ascertain only the spacing and rate of change in spacing between the following vehicle and the second lead vehicle.

It is assumed that for every decision interval, the following vehicle driver sets a unique "comfort zone". The comfort zone defines the spacing the following vehicle driver desires between his/her own vehicle and each of the two lead vehicles. The length of the comfort zone is assumed to depend on the current speed of the following vehicle. If the current spacing is shorter than that dictated by the driver's comfort zone and is decreasing in length, the following vehicle driver will decelerate to increase the spacing. Conversely, if the current spacing exceeds that set by the driver’s comfort zone, and the vehicle is travelling at a speed below the control speed, the following vehicle driver will accelerate.

As shown in Figure 1, the proposed car-following model assumes that the level of alertness of a driver affects the perception/reaction time component of the acceleration/deceleration rate. A number of researchers, for example, Ozaki (6), Johansson and Rumar, (15), Olson and Sivak (16), Racklet et al. (17), Sivak et al. (18), and Diew et al. (19) reported that if a driver is alert, less time is needed to perceive and react to a given situation. It is assumed that the following vehicle driver will modify his or her perception/reaction time with respect to his/her level of alertness. The level of alertness is assumed to depend upon current spacing, desired spacing between the following vehicle and each of the lead vehicles, and the status of the lead vehicles' brake lights. It is assumed that the following vehicle driver becomes more alert with reduced perception/reaction times when brake lights of the lead vehicle(s) are on and the lead vehicle(s) is/are within the following vehicle driver's comfort zone.

The rigorous framework for formulating relationships among different variables of the proposed car-following model with their calibration and validation is presented in the following section.

Model Formulation

Following the underlying assumptions for the proposed car-following model, the stock flow diagram of the model was developed using System Dynamic methodology and ITHINK© software platform. Figure 2 shows the stock-flow diagram of the proposed car-following model. As illustrated in Figure 2 the proposed car-following model consists of two sectors, speed and spacing. Functions in each sector interact with functions in the other sectors through feedback links. The first and second lead vehicle speed profiles are specified externally and prescribe the lead vehicle target conditions for input into the following vehicle speed and spacing sectors. The acceleration/deceleration rate, speed and spacing of the following vehicle are determined within the model, subject to rules and assumptions prescribed in following sections.

Acceleration/deceleration rate

Unlike many existing car-following models, in the proposed car-following model the absolute speed and/or acceleration of the lead vehicles are not required as inputs in setting the following vehicle acceleration/deceleration rates and spacing. This assumption differs from many existing car-following models and can be viewed as being more parsimonious than existing models in estimating the following vehicle's acceleration/deceleration rate.

In the proposed car-following model the acceleration/deceleration rate of the following vehicle at simulation time $t$ is determined as follows:

$$a^F(t) = \left[ \frac{V^F_c(t) - V^F(t)}{PRT^F(t)} \right] \times 0.278$$  \hspace{1cm} (1)
Where,

\[ a^F(t) = \text{Acceleration/deceleration rate of the following vehicle at time } t \text{ (m/sec}^2)\].

\[ V_{CF}^F(t) = \text{Control speed of the following vehicle at time } t \text{ in steady and unsteady state (Km/h).} \]

\[ V^F(t) = \text{Current speed of the following vehicle at time } t \text{ (Km/h).} \]

\[ \text{PRT}^F(t) = \text{Perception reaction time of the following vehicle driver at time } t \text{ (sec).} \]

\[
0.278 = \text{Unit conversion factor, for converting Km/h to m/sec.}
\]

It is assumed that the following vehicle driver attempts to match his/her actual speed \( V^F \) with the control speed \( V_{CF}^F \) at a given spacing and rate of change in spacing with respect to the lead vehicle immediately in front of him/her. The control speed \( V_{CF}^F \) refers to a maximum achievable speed by the following vehicle driver in both steady and unsteady conditions. The steady state conditions represent the situations when relative speed between the following and the lead vehicle is zero, while unsteady state conditions are those when relative speed between the following and the lead vehicle is not zero. The control speed \( V_{CF}^F \) is calculated using equation 2.

\[
V_{CF}^F(t) = V_{CS}^F(t) \times \alpha(t)
\]

Where,

\[
V_{CF}^F(t) = \text{Control speed of the following vehicle in steady and unsteady state at time } t \text{ (Km/h).}
\]

\[
V_{CS}^F(t) = \text{Control speed of the following vehicle in steady state at time } t \text{ (Km/h).}
\]

\[
\alpha(t) = \text{Effect of rate of change in spacing between the following and the second lead vehicle on } V_{CF}^F \text{ at time } t \text{ (dimensionless).}
\]

The control speed of the following vehicle in steady state conditions \( V_{CS}^F \) is assumed to depend on current spacing between the following and the second lead vehicle \( S^F \). The relationship between \( V_{CS}^F \) and \( S^F \) is calibrated based on observed individual vehicle tracking data obtained from the SAVME database. The SAVME database provides a complete microscopic record of trajectories and distance headways observed for individual vehicles in a traffic stream over a period of time. The database contains 18 hours of vehicle trajectory data representing over 30,500 vehicles traversing a 152m segment of a 5-lane arterial street in Ann Arbor, Michigan. The road segment includes a 4-leg intersection that is stop controlled on the minor street approaches. All data were collected during weekday daylight hours during non-inclement weather conditions (20).

Trajectory data for a random sample of 164 vehicle pairs were extracted from the SAVME database. For each pair of vehicles, the speed of the following vehicle and the spacing were extracted. For each observed speed, the mean distance headway (spacing) from all vehicles observed to travel at this speed was computed. The results are illustrated in Figure 3 as the control speed versus observed mean spacing. To ensure realistic behavior at the boundaries of the relationship shown in Figure 3, constraints are incorporated such that the control speed must be non-negative and not greater than the maximum assumed speed of 80 Km/h. Based on field data observations it is assumed that for \( S^F \geq 45 \text{ m} \) the \( V_{CS}^F \) would be 80 Km/h, and for \( S^F \leq 9 \text{ m} \) the \( V_{CS}^F \) would be 0 that yields a jam density of 110 Vehicles/Km. The relationship illustrated in Figure 3, and defined in Equation 3, is consistent with the data obtained from a Newcastle University research team in the United Kingdom (21). Similar to the SAVME database, the Newcastle data tends to demonstrate a fairly aggressive car-following behavior at short spacing and less aggressive car-following behavior at longer spacing, as illustrated by Figure 3.

\[
V_{CS}^F(t) = \begin{cases} 
80 \text{ Km/h} & \text{if } S^F(t) \geq 45 \text{ m} \\
43.46 \times \ln(S^F(t)) - 83.3 & \text{if } 9 \text{ m} < S^F(t) < 45 \text{ m} \\
0 & \text{if } S^F(t) \leq 9 \text{ m}
\end{cases}
\]

(3)
Where,

\[ S_F^F(t) = \text{Current spacing between the following and the second lead vehicle at time } t \text{ (m)}, \text{ defined as:} \]

\[ S_F^F(t) = S_F(t) - [V_F^F(t) - V_{L2}^F(t)] \times dt \]  \hspace{1cm} (4)

Where,

\[ S_F(t-dt) = \text{Current spacing between the following and the second lead vehicle at time } t-dt, \]
\[ \text{initially at time } t = 0 \text{ it is externally defined (m).} \]
\[ dt = \text{simulation interval, assumed } dt = 0.1\text{sec.} \]
\[ V_F^F(t) - V_{L2}^F(t) = \text{Rate of change in spacing between the following and the second lead vehicle at time } t \text{ (m/sec).} \]

Observations in the SAVME database for the relationship in equation (3) suggest that the control speed \( V_{CS}^F \) for a given spacing differs among drivers. This is likely due to differences in age, gender, risk taking propensity, skills, and vehicle performance characteristics. Moreover, the situational factors such as time of day, day of week, road geometry, traffic conditions, weather and road conditions also influence the control speed of a driver for a given spacing. The proposed car-following model assumes ideal roadway conditions and does not explicitly consider individual driver differences or situational factors.

In equation (2), \( \alpha \) is assumed to depend on rate of change in spacing between the following and the second lead vehicle. In the absence of sound empirical evidence, different relationships (shown in Figure 4) were investigated to find the best relationship for \( \alpha \). The procedure for investigating the best relationship for \( \alpha \) is discussed later in this paper. As noted by Legasto et. al. (22) the heuristic nature of the relationships, such as that illustrated in Figure 4, is not a source of weakness of SD applications but rather a source of strength. Through these types of functions a SD model is able to represent essential phenomena which might otherwise be omitted from lack of sound empirical evidence. This does not mean that these relationships should be purely subjective in nature but rather that they should be initially defined and incorporated into the process, and later investigated in more depth using whatever empirical evidence is available.

In Figure 4, the horizontal axis represents the normalized values of rate of change in spacing (NRS\( S_F^F \)), while the vertical axis represents the assumed values for the effect of rate of change in spacing on control speed (\( \alpha \)). The normalized values for rate of change in spacing are defined as the rate of change in spacing divided by the current spacing between the lead and following vehicle at time \( t \) (equation 5). It is assumed that as the rate of change in spacing decreases (i.e. vehicles are getting closer to each other), drivers will reduce their control speed in response to increased risk of collision. In Figure 4 only situations where vehicles get closer to each other (i.e. only negative values of rate of change in spacing) are considered, since these are more critical for increased risk.

\[ \text{NRS}_F^F(t) = \frac{[V_F^F(t) - V_{L2}^F(t)]}{S_F^F(t)} \]  \hspace{1cm} (5)

Where,

\[ \text{NRS}_F^F(t) = \text{Normalized values of rate of change in spacing between the following and the second lead vehicle at time } t \text{ (1/sec).} \]

In equation (1), speed of the following vehicle at time \( t \) (\( V_F^F(t) \)) is determined by using the following equation.
\[
V_F^F(t) = V_F^F(t - dt) + a_F^F(t - dt) \times 3.6 \times dt
\]  
(6)

Where,
- \( V_F^F(t-dt) \) = Speed of the following vehicle at time \( t-dt \), initially at \( t = 0 \) it is externally defined (Km/h).
- \( a_F^F(t-dt) \) = Acceleration/deceleration rate of the following vehicle at time \( t-dt \), calculated using equation (1) at time \( t-dt \) (m/sec\(^2\)).
- 3.6 = Unit conversion factor, for converting m/sec to Km/h.

The mathematical formulation for perception reaction time of the following vehicle driver at time \( t \), \( PRT_F^F(t) \), in equation (1) is discussed in the next section.

**Perception reaction time of the following vehicle driver**

A number of researchers, for example, (15-19) experimentally measured the perception-reaction time of drivers in anticipated and unanticipated traffic conditions. These researchers found that in anticipated traffic conditions, such as, stopped traffic at an intersection because of a red traffic signal, drivers become more alert and they require less time to perceive and react to such situations. However, majority of the existing car-following models make a simplifying assumption that the perception-reaction of drivers in both anticipated and unanticipated traffic conditions is constant. The proposed car-following model incorporates the variation in perception-reaction times of drivers based on changes in traffic conditions. It is assumed that the following vehicle driver becomes more alert with reduced perception-reaction time when the lead vehicle(s) brake lights are lit and the lead vehicle(s) is/are within the following vehicle driver's comfort zone.

In the proposed car-following model the perception-reaction time of the following vehicle is determined by the following equation.

\[
PRT_F^F(t) = \frac{NPRT_F^F(t)}{[A^{L2}(t) \times A^{L1}(t)]}
\]  
(7)

Where,
- \( NPRT_F^F(t) \) = Normal perception-reaction time of the following vehicle driver at time \( t \) (sec).
- \( A^{L2}(t) \) = Alertness of the following vehicle driver due to the second lead vehicle at time \( t \) (dimensionless).
- \( A^{L1}(t) \) = Alertness of the following vehicle driver due to the first lead vehicle at time \( t \) (dimensionless).

The level of alertness of the following vehicle driver is defined as:

\[
A^{L2}(t) = \begin{cases} 
\beta 2(t) & \text{brake lights of second lead vehicle at time } t \text{ are LIT} \\
1.0 & \text{otherwise}
\end{cases}
\]  
(8)

\[
A^{L1}(t) = \begin{cases} 
\beta 1(t) & \text{brake lights of second lead vehicle at time } t \text{ are LIT} \\
1.0 & \text{otherwise}
\end{cases}
\]  
(8)
Where,

\[ \beta_2(t) = \text{Effect of change in ratio of current to desired spacing between the following and the second lead vehicle on perception reaction time of the following vehicle driver at time } t \text{ (dimensionless).} \]

\[ \beta_1(t) = \text{Effect of change in ratio of current to desired spacing between the following and the first lead vehicle on perception reaction time of the following vehicle driver at time } t \text{ (dimensionless).} \]

The status of the lead vehicles' brake lights in equation (8) and (9) can be ascertained internally based on the following relationship suggested by Ozaki (6) between speed and deceleration rate of the lead vehicle. However, in real traffic situations a driver does not need the following expressions to determine the status of lead vehicle's brake lights, as he/she can ascertain the brake light status of lead vehicles visually.

\[
\begin{align*}
\text{if } d_{L2}(t) &< -0.013 \times V_{L2}(t) \text{ then brake lights at time } t \text{ are ON else Off } \\
\text{if } d_{L1}(t) &< -0.013 \times V_{L1}(t) \text{ then brake lights at time } t \text{ are ON else Off }
\end{align*}
\]

(10)

Where,

\[ d_{L2}(t), d_{L1}(t) = \text{Deceleration rate of the second and the first lead vehicle at time } t \text{ respectively.} \]

\[ V_{L2}(t), V_{L1}(t) = \text{Current speed of the second and the first lead vehicle at time } t \text{ respectively.} \]

The proposed car-following model assumes that the perception-reaction of the following vehicle driver depends on his/her level of alertness. Alertness is defined in terms of a dimensionless quantity that modifies the driver's perception-reaction time with respect to brake lights status of the lead vehicles. It is assumed that at simulation time \( t \), the alertness of the following vehicle driver would be either equal to 1.0 or the product of \( \beta_1(t) \) and \( \beta_2(t) \) depending upon the status of the brake lights of the lead vehicles and their positions with respect to the comfort zone of the following vehicle.

It is assumed that for a level of alertness value of 1.0 the perception reaction time of the following vehicle driver (\( \text{PRT}^F \)) would be equal to his/her normal perception-reaction time (\( \text{NPRT}^F \)). The \( \text{NPRT}^F \) represents the total time it takes a driver to perceive an object or target and initiate the action in normal situations or unanticipated traffic situations. In the proposed model \( \text{NPRT}^F \) is assumed to be 2.0 sec and it is modified based on values of \( \beta_1 \) and \( \beta_2 \). Like the relationship for \( \alpha \) illustrated in Figure 4, different relationships for \( \beta_1 \) and \( \beta_2 \) shown in Figure 5 were investigated to find the best relationship for \( \beta_1 \) and \( \beta_2 \). The procedure for investigating the best relationship for \( \beta_1 \) and \( \beta_2 \) is discussed later in this paper. The proposed model assumes the same relationship for \( \beta_1 \) and \( \beta_2 \). The only difference is that for \( \beta_2 \) the horizontal axis of Figure 5 represents the ratio of current to desired spacing between the following and the second lead vehicle (\( \text{SR}^{l2} \)), while for \( \beta_1 \) the horizontal axis of Figure 5 represents the ratio of current to desired spacing between the following and the first lead vehicle (\( \text{SR}^{l1} \)). The vertical axis of Figure 5 for both \( \beta_1 \) and \( \beta_2 \) represents assumed effect of change in spacing ratio on the perception-reaction time of the following vehicle driver. It is assumed that as the \( \text{SR}^{l2} \) or \( \text{SR}^{l1} \) decreases (i.e. current spacing less than the desired spacing of the following vehicle driver), the level of alertness of the following vehicle driver would rise to its maximum assumed value. The boundary limits on the vertical axis of Figure 5 are set so as to satisfy the extreme limits of a driver's perception-reaction time in alerted situations as reported by different researchers (15-19).

\[ \text{SR}^{l2} \text{ and } \text{SR}^{l1} \text{ at time } t \text{ are determined by the following equations.} \]

\[ \text{SR}^{l2}(t) = \frac{S^F(t)}{DS^F(t)} \]

(11)
(t)DS = S(t) F (t)

Where,

- SR\text{L1}(t) = \frac{S(t)}{DS_{1F}(t)}
- SR\text{L2}(t) = \text{Ratio of current to desired spacing between the following and the second lead vehicle at time } t \text{ (dimensionless)}.
- S\text{F}(t) = \text{Current spacing between the following and the second lead vehicle at time } t \text{ (m)}.
- DS_{2F}(t) = \text{Length of comfort zone or desired spacing between the following and the second lead vehicle at time } t \text{ (m)}.
- SR\text{L1}(t) = \text{Ratio of current to desired spacing between the following and the first lead vehicle at time } t \text{ (dimensionless)}.
- DS_{1F}(t) = \text{Length of comfort zone or desired spacing between the following and the first lead vehicle at time } t \text{ (m)}.
- S(t) = \text{Current spacing between the following and the first lead vehicle at time } t \text{ (m)}, \text{ and defined as:}

\[ S(t) = S\text{F}(t) + S^2(t) \]  \hspace{1cm} (13)

Where,

- S^2(t) = \text{Current spacing between the second and the first lead vehicle at time } t \text{ (m)}, \text{ and defined as:}

\[ S^2(t) = S^2(t - dt) + [V_{L2}(t) - V_{L1}(t)] \times dt \]  \hspace{1cm} (14)

Where,

- S^2(t - dt) = \text{Current spacing between the second and the first lead vehicle at time } t - dt, \text{ initially at time } t = 0 \text{ it is externally defined (m)}.
- V_{L2}(t) - V_{L1}(t) = \text{Rate of change in spacing between the second and the first lead vehicle at time } t \text{ (m/sec)}.

The next section describes how the comfort zone of the following vehicle driver with respect to the second and the first lead vehicle is determined.

**Comfort zone**

The comfort zone defines the spacing the following vehicle driver desires between his/her own vehicle and each of the two lead vehicles. The length of the comfort zone is a function of the speed of the following vehicle and defined in the following.

\[ DS_{2F}(t) = \begin{cases} 9 \text{ m} & 0 \leq V\text{F}(t) \leq 20 \text{ Km/h} \\ 7.0004 \times e^{(0.0024 \times V\text{F}(t))} & 20\text{Km/h} < V\text{F}(t) < 80\text{Km/h} \\ 45 \text{ m} & V\text{F}(t) \geq 80 \text{Km/h} \end{cases} \]  \hspace{1cm} (15)

Where,

- DS_{2F}(t) = \text{Length of comfort zone or desired spacing between the following and the second lead vehicle at time } t \text{ (m)}.
- V\text{F}(t) = \text{Speed of the following vehicle at time } t \text{ (Km/h)}.

The equation (15) is the mirror image of the relationship between control speed and spacing given in equation (3). The calibration of relationship defined in equation (15) is based on the same sample of 164
vehicle used for calibration of relationship defined in (3). The relationship defined in equation (15) is also depicted in Figure 6. To ensure realistic behavior at the boundaries of the relationship shown in Figure 6, constraints are incorporated based on field data observations such that for $V_F \geq 80$ Km/h the DSF should be 45m, and for $DS_F = 0$ Km/h the DSF should be 9 m.

The length of the comfort zone of the following vehicle driver with respect to the first lead is assumed to be twice the length of comfort zone with respect to the second vehicle, and defined as:

$$DS_1^F(t) = 2 \times DS_2^F(t) + L$$

(16)

Where,

$$DS_1^F(t) = \text{Length of comfort zone or desired spacing between the following and the first lead vehicle at time } t, \text{ (m).}$$

$$L = \text{Length of a vehicle (m).}$$

Before investigating the best relationships for $\alpha$, $\beta_1$, and $\beta_2$, the evaluation of the proposed model was carried out using assumed curve C in Figure 4 for $\alpha$ and curve 1 in Figure 5 for $\beta_1$ and $\beta_2$. The next section describes the evaluation of the proposed car-following model.

EVALUATION OF PROPOSED CAR-FOLLOWING MODEL

The microscopic evaluation of the proposed car-following model is conducted by comparing model estimates of speed and spacing for the following vehicle to those observed in the SAVME database. Fifty samples of three-vehicle strings were randomly selected from SAVME database. For each sample the trajectories of the following and both lead vehicles were extracted from the SAVME database. The trajectory of the first and second lead vehicle, and the initial speed and position of the first lead, second lead, and the following vehicles were provided as inputs to the proposed car-following model. The model was then used to estimate the behavior of the following vehicle in response to the known behavior of the first and the second lead vehicle. The estimated behaviour of the following vehicle driver in fifty experiments is discussed in the following sections.

Qualitative evaluation of the proposed car-following model

Qualitative evaluation of the proposed car-following model includes comparing the model predicted dynamic pattern to the observed dynamic pattern. Figure 7 illustrates the observed and model predicted dynamic patterns for three different samples randomly selected from 50 samples used for evaluation of the proposed car-following model. Figure 7 demonstrates observed and predicted speed and spacing profiles of the following vehicle. As indicated by the results illustrated in Figure 7, the speed and spacing profiles predicted by the proposed car-following model closely follow those in the observed field data.

Quantitative evaluation of the proposed car-following model

For each of 50 samples the root-mean-squared (RMS) error associated with the prediction of speeds and spacing of following vehicle was estimated as given in Table 1. The average RMS error associated with the prediction of following vehicle speed and spacing for the fifty samples was found to be 2.55 Km/h and 1.79 m respectively.

A regression analysis of predicted and observed mean speed and mean spacing of the following vehicle was carried out for the sample application. The results are shown in Figures 8 and 9. Figure 8 shows the plot of predicted versus observed mean speed of the following vehicle. Figure 9 shows the plot of predicted versus observed mean spacing of the following vehicle. The results indicate significant agreement between the predicted output from the model and the observed field data. Based on comparison between the
proposed car-following estimates and observed SAVME data, it is suggested that the proposed model can closely reflect observed speed and spacing profiles for selected three-vehicle strings, where following vehicle drivers consider two lead vehicle stimuli in setting speeds and spacing over time.

INVESTIGATION OF BEST RELATIONSHIPS FOR $\alpha$, AND $\beta^2$

Investigation of the best relationships for $\alpha$ and $\beta^2$ is carried out simultaneously using 35 samples extracted from the SAVME database. In the absence of sound empirical evidence, the proposed car-following model discussed above was used to find the best relationships for $\alpha$ and $\beta^2$. The proposed model was applied to each of 35 the samples for all possible combinations of relationships depicted in Figure 4 and 5 (i.e. total simulation runs = 35 * 3 * 3 = 315). For each simulation run the RMS error between model predicted and observed following vehicle speed was calculated. Table 2 summarizes the average RMS error for these simulation runs. These results indicate that the combination of relationship 1 from Figure 5 and C from Figure 4 yields the lowest RMS error (3.33). The average frequency of minimum RMS error for all possible combinations of relationships was also calculated (Table 3). The results indicate that the combination of relationship 1 and C yields minimum RMS error for maximum number of simulation runs (31%).

Based on the results shown in Tables 2 and 3, relationship C is selected as the best relationship for $\alpha$, and relationship 1 as representing the best relationship for $\beta^2$. These are coincidentally the same assumed relationships used for evaluation of the proposed car-following model. It is acknowledged that the procedure adopted for calibrating the above relationships is not very robust statistically. However, based on available observed data and an intuitive or heuristic understanding of these relationships, relationships 1 and C appear to provide the most reasonable explanation of driver behaviour.

CONCLUSIONS

In this paper we have discussed a number of existing car-following models and have identified several common shortcomings. We have presented a revised car-following model based on System Dynamics principles, which attempts to address many of these shortcomings. The proposed model assumes that drivers adjust their speed based on the current spacing and rate of change in current spacing to the next downstream vehicle. The model also takes into account the driver's control speed and distance headway in relation to increased risk of collisions.

The proposed model assumes that drivers are capable of estimating the spacing between their own vehicle and the next downstream vehicle. The model, unlike many existing car-following models, does not make unrealistic assumptions about drivers' ability to estimate the absolute speed of the downstream vehicles.

In this paper we have compared the model estimates of speed and spacing profiles for the following and second lead vehicle to the speed and spacing profiles of observed vehicles. These comparisons suggest that the proposed car-following model yields realistic results in replicating the behavior of the following vehicle driver from an observed vehicle tracking database. In the proposed model drivers seek to maintain the speed and spacing that is consistent with their understanding of the risks involved for any traffic situation.

ACKNOWLEDGEMENTS

The authors are grateful to the researchers at UMTRI, and in particular Drs. Robert Ervin and Jeff Walker, for providing the SAVME database.
REFERENCES

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TABLE 1 RMS Error Associated with Predicting Following Speed and Spacing based on 50 sample

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<table>
<thead>
<tr>
<th>Sample</th>
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<th>RMS Error (Km/h)</th>
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** Each observation represents one deci second of time.
TABLE 2 Average RMS Error Associated with Predicting Following Vehicle Speed based on 35 Observed Cases.

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<th>Effect of change in spacing on perception Reaction time</th>
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### TABLE 3 Average Frequency of Minimum RMS based on 35 Observed Cases

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LIST OF FIGURES:

FIGURE 1 Underlying assumptions of the proposed car-following model.

FIGURE 2 Stock-flow diagram of proposed car-following model.

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FIGURE 4 Hypothesized relationships between rate of change in spacing and its effect on control speed.

FIGURE 5 Hypothesized relationships between ratio of spacing and its effect on perception reaction time.

FIGURE 6 Relationship between $V^F$ and $DS^F$.

FIGURE 7 Comparison of predicted verses observed speeds and spacing of following vehicle (Three data sets).

FIGURE 8 Predicated versus observed mean speed of following vehicle based on 50 cases.

FIGURE 9 Predicated versus observed mean spacing of following vehicle based on 50 cases.
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FIGURE 2 Stock-flow diagram of proposed car-following model.
FIGURE 3 Calibrated relationship between $V_{CS}^F$ and $S^F$.

$y = 43.457 \ln(x) - 83.297$

$R^2 = 0.9752$
FIGURE 4 Hypothesized relationships between rate of change in spacing and its effect on control speed.

FIGURE 5 Hypothesized relationships between ratio of spacing and its effect on perception reaction time.
FIGURE 6 Relationship between $V^F$ and $D^F$.

\[ y = 7.0004e^{0.0224x} \]

\[ R^2 = 0.9752 \]
FIGURE 7 Comparison of predicted verses observed speeds and spacing of following vehicle (Three data sets).
FIGURE 8 Predicated versus observed mean speed of following vehicle based on 50 cases.

\[ y = 0.9725x + 1.2097 \]
\[ R^2 = 0.9899 \]

FIGURE 9 Predicated versus observed mean spacing of following vehicle based on 50 cases.

\[ y = 0.9908x + 0.5232 \]
\[ R^2 = 0.9847 \]