Fuzzy Preference Based Route Choice Model

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Abstract:
This paper introduces a methodology for route choice based on fuzzy preference relations. The core of the model is FiPV (Fuzzy-individuelle Präferenzen von Verkehrsteilnehmern or fuzzy traveler preferences), that is a choice function based on fuzzy pairwise comparisons for travel decisions. The proposed model may be the first application of fuzzy individual (preference-based) choice in travel demand modeling and probably also the first in this class of fuzzy models to consider the spatial knowledge of individual travelers in route choice. It is argued that travelers do not or cannot always follow perfect maximization principle. We formulate therefore a model that also takes into account the travelers with non-perfect-maximizing behavior. Although the model is not yet supported by empirical evidence, it shows a more transparent structure than those of the conventional dynamic route choice.

Keywords:
route choice behavior, fuzzy preference relations, bounded rationality, inductive reasoning, preference updating, dynamic traffic assignment, information, ITS.

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Introduction

Route choice is an essential part of traffic assignment model. The most existing route choice models are random utility models with perfect rational assumption that all feasible paths are available for individuals. More recently, the new functional specifications of route choice models and explicit modeling of path choice set formation have been addressed. Researchers today work more intensively on the specification of complex models of behavior and traffic movement. New insights can be gained about network behavior by examining the travelers’ behaviors and cognitive processes underlying them in detail. There is also a clear indication that travel time and cost do not always dominate the route selection process. These could have important implications to the future development of route choice models.

The original motivation for the development of a fuzzy preference based route choice model was to more realistically represent traveler’s decision making, esp. in response to ITS. The fundamental element of the proposed model is a concept named FiPV (Fuzzy-individuelle Präferenzen von Verkehrsteilnehmern or fuzzy traveler preferences) which is concerned with discrete decision problems provided that pairwise comparisons between alternatives are available with inherent subjectivity and imprecision of human thinking.

We focus our study only on the structure of route choice behavior under FiPV. One contribution of this research is the introduction of fuzzy preference based choice in travel demand analysis. Another contribution is the new methodology for route choice. We will first review the literature concerning previous research in fuzzy route choice and then we present the model formulation. An academic example will then follow.

Review of the previous research on fuzzy route choice

Research in the field of soft computing has been exploring the application of fuzzy set theory as a framework within which many transportation problems can be studied. While several researchers have demonstrated the applicability of fuzzy logic to traffic control and management tasks (Zimmermann, 1999), application to traffic modeling in itself has remained a relatively unexplored topic (see also Hoogendoorn et al. (1998) and Teodorovic (1999) for comprehensive review).

Teodorovic and Kikuchi (1990) were first to model the complex route choice problem using fuzzy logic. They used fuzzy inference techniques to study the binary route choice problem. Akiyama et al. (1993) developed a model for route choice behavior based on the fuzzy reasoning approach. Lotan and Koutsopoulos (1993) developed models for route choice behavior in the presence of information based on concepts from approximate reasoning and fuzzy control. Continuing their research, Vythoulkas (1994) introduced the concept of rule
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weight to represent the different importance that decision makers attribute to different rules, and also proposed a new calibration procedure based on the theory of neural networks.

Henn (2000) suggested a fuzzy version of a deterministic choice model and proves that fuzzy route choice model is a generalization of the standard logit model, in which the modeling approach is seen as a general framework of random utility based models. The effect of information on drivers is modeled as a modification of the imprecision or the uncertainty of the predicted cost of a route. Peeta and Yu (2002) presented a fuzzy logic and control based model to predict on-line driver VMS route diversion decisions. It is done by transforming the probabilities obtained from the data to possibilities.

Zhao (1994) proposed a new concept of user equilibrium: the $\varepsilon$-equilibrium. Drivers perception of generalized travel time is modeled using a fuzzy number. Based on the perceived generalized travel times of the different route alternatives, drivers will choose a route which optimizes this fuzzy travel time. Wang and Liao (1999) focused their study on solving a user equilibrium problem in traffic assignment when the node-arc incidence matrix is fuzzy. By considering it as a variational inequality problem with fuzzy function in a convex cone, this problem is reformulated into a multiple objective programming model. Henn (1999) developed a route choice model based on possibility measure and defined a fuzzy user equilibrium (FUE), an extended UE for dynamic systems into near-equilibrium state. FUE means that “each used route has approximately the same cost and it is almost minimal.” Chang and Chen (2000) employed the variational inequality approach to formulate a link-based fuzzy user-optimal route choice problem embedding link interactions.

Most of these models employ perfect maximization principle. If the travelers are not utility-maximizers, then such models have a less meaning. In this research we propose a model that represents decision structure in route choice, which does not only rely on optimization criteria such as shortest path and least time, but applies fuzzy preference relations to yield the actual choice of the travelers.

Structure of the FiPV model

Route Choice and Network Notation

Factors that influence route choice typically relate to one of four categories (Bovy & Stern, 1990):

- the perceived available routes,
- the character of the traveler,
- the trip to be made, and
- other circumstances.

Moreover, travelers commonly trade-off travel time to select routes that are more direct, beautiful landscape, free (non-toll road) or safer alternative. Route selection decisions of traveler often depend on what other travelers are doing. Route choice can depend on congestion caused by the aggregated behavior of others. This may cause a standstill, since nobody can choose a route because no one knows what everybody else will do. But travel decision (route choice) must be made.

Set of alternatives. The problem of alternatives perception is particularly acute in route choice where hundreds of alternatives are potentially available for large networks. On the other hand,
available empirical evidence shows that only a limited number of paths are actually perceived by travelers (Cascetta et al., 1996; Golledge, 1997). The set of alternatives is normally unobserved or latent. We use some research results that can give a foundation to study real available alternatives faced by the travelers.

**Network notations.** We adopt a network model with multiple origins and destinations. Our transportation network is represented by a directed graph with nodes and links. Let $\mathcal{G}=(\mathcal{N},\mathcal{A})$ be a transport network, where $\mathcal{N}$ is the set of nodes, corresponds to origin, destination, intersection of number of streets, railway station, airport, etc. and $\mathcal{A}$ is the set of directed links, corresponds to streets, public transport lines, etc. In this paper we consider only the road network. Let $R (R \subseteq \mathcal{N})$ denote the set of origins node $r$ and $S (S \subseteq \mathcal{N})$ denote the set of destinations node $s$. Each O-D pair $r-s$ is connected by a set of routes (paths) through the network. This set is denoted by $P^{rs}$. A path $p$ ($p \in P^{rs}$) is a sequence of adjacent directed links, all distinct, that begins at an origin $r$ and ends at a destination $s$. Consider the time period $[0,T]$, which is enough to allow all travelers departing during the day to complete their trips. The fixed period $[t_r,t_s]$ is a time, which the individual traveler $n$ needs to travel from origin $r$ to destination $s$. Let $x_a$ and $t_a$ represent the flow and travel time on link $a$ (where $a \in \mathcal{A}$); $t_a(\cdot)$ represents the relationship between flow and travel time for link $a$, thus $t_a = t_a(x_a)$ is the link performance function. Similarly, let $f_p^{rs}$ and $c_p^{rs}$ represent the flow and travel time, respectively, on path $p$. The travel time on a particular path is the sum of the travel time on the links comprising the path. This relationship can be expressed as

$$c_p^{rs} = \sum_a t_a \delta_{ap}^{rs} \quad \forall p, r, s \quad (1)$$

where $\delta_{ap}^{rs}=1$ if the link $a$ is part of path $p$, and $\delta_{ap}^{rs}=0$ otherwise. Let

$$x_a(t) = \text{number of vehicles traveling on link } a \text{ at time } t;
$$

$$x_{ap}^{rs}(t) = \text{number of vehicles traveling on link } a \text{ over route (path) } p \text{ with origin } s \text{ and destination } s \text{ at time } t.$$ 

It follows that

$$x_a(t) = \sum_{rsp} x_{ap}^{rs}(t) = \sum_{rsp} f_p^{rs} \delta_{ap}^{rs} \quad \forall a \quad (2)$$

Let $u_a(t)$ denote the inflow rate (vehicles/hour or vehicles/minute) into link $a$ at time $t$ and $v_a(t)$ exit flow rate from link $a$ at time $t$. These inflow and outflow on link $a$ are both regarded as control variable. The number of vehicles on link $a$ is defined as the state variable for link $a$, that can be written as

$$\frac{dx_{ap}^{rs}(t)}{dt} = u_{ap}^{rs}(t) - v_{ap}^{rs}(t) \quad \forall a, p, r, s \quad (3)$$

**Travel times.** The instantaneous link travel time $c_a(t)$ at any time $t$ is defined as the travel time that would be experienced by vehicles traversing a link when prevailing traffic conditions remain unchanged. The instantaneous path travel time at any time $t$, $\psi_p^{rs}(t)$, is defined as the sum of the instantaneous link travel time over all link in route $p$, that can be expressed as
Define $\psi^T_p(t) = \sum_{a \in \text{trp}} c_a[x_a(t), u_a(t), v_a(t)] = \sum_{a \in \text{trp}} c_a(t)$ \quad $\forall r, s, p$ (4)

Effect of familiarity and travelers’ knowledge of the transportation network. The rational behavior for selecting a path might be based on geographic network layout, road features and traffic features. Among these factors, spatial ability and knowledge of the transportation network play an important role in explaining the variation in routes selected. Drivers who have a greater familiarity with network conditions and network layout should be expected to make more efficient pre-trip route choices as well as be able to better adjust their travel pattern en route in the presence of severe congestion. Experiments by Adler et al. (1993) suggest that previous experiences with the current and alternate routes influence route switching. The results obtained by Lotan (1997) indicate larger homogeneity among the unfamiliar drivers in terms of their switching and diverting behavior, while familiar drivers demonstrate larger taste and preference variations. Ramming (2002) proposes a model of network knowledge and its coefficient estimates, based on the geographical idea of spatial ability. He shows that travelers with more network knowledge appear to vary their commute route to respond to changing travel conditions. These provide a basis of our conception about the route choice process of the travelers.

Eigen-network. Let us introduce a concept of Eigennetzwerk (eigennetwork). Travelers cannot have a perfect knowledge of the network, because of their limited cognitive capacity. A cognitive capacity of a traveler $n$ is therefore not unbounded. Accordingly, in the traveler’s mental map can be detected a unique network called eigennetwork, which contains the set of his own cognitive nodes and links. This eigennetwork is extremely situational, i.e. depending on the actual network performance, and denoted by $\mathcal{G}_n = (\mathcal{N}_n, \mathcal{A}_n)$, $\mathcal{G}_n \in \mathcal{G}$.

Three-stage decision making. In this study, a three-stage decision making process in selecting routes on networks will be proposed, namely: network recognition, selection the global decision nodes, and final decision activity within smallest decision segment. Other researchers e.g. Hato and Asakura (2000) suggested the same structure. The first stage of decision process provides a cognitive recognition of the network. How a traveler recognizes the existing transportation networks has been explored by psychologists and geographers, we adopted a concept of mental map as mentioned. In the second stage, decision nodes will be identified and therefore the smallest decision segment could be determined. This process presents a course of behaviors to eliminate impractical routes from the recognized networks based on a certain reference value and to pick out the alternative routes that are considered for selection. This current paper is more concerned with the third stage, specifically, the application of FiPV in decision activity to choose the routes. The travelers are not assumed to compare the utilities among allowable route alternatives and then to select a final route of the highest utility, but rather they are assumed to be using their own fuzzy preferences in selecting a path.
**Decision nodes.** To address the modeling of en route guidance acquisition and path switching, a subset $B \subseteq \mathcal{N}$ of nodes will be described as decision nodes (cf. Bottom, 2000; Adler et al., forthcoming). At these nodes, trips may change from one path to a different one continuing on to the same destination; such changes are not otherwise possible. The set of decision nodes encountered on paths going from origin $r$ to destination $s$ at time $t$ is designated $B_{ap}^r(t)$. Origin $r$ is always considered to be a decision node. Trips entering the network receive information there. If these are the only decision nodes, then only pre-trip path choice is allowed. Alternatively, if all nodes are decision nodes, then path switches are allowed anywhere.

Decision nodes induce a natural decomposition of paths into sub-paths: an arbitrary sequence of adjacent directed links that begins at decision node $b$ and ends at decision node $e$ (see Figure 1). A complete origin-destination path can be seen as a sequence of adjacent sub-paths connected at decision nodes. Given a decision node $b \in B_{ap}^r(t)$, $P_b^r(t)$ designates the set of complete paths from $r$ to $s$ that go through $b$, $P^r_e(t)$ designates the set of sub-paths going from $r$ to $s$ at time $t$ and $P_b^s(t)$ designates the set of all sub-paths going from $r$ to $s$ at time $t$.

![Figure 1 Structure of travel path choice within smallest decision segment $m$ (Routenentscheidungsabschnitt)](image)

**Decision segment.** Network recognition sets up the eigennetwork $\mathcal{S}_n$ of which the traveler selects, in his own capacity, the possible alternative routes for a travel from origin $r$ to destination $s$. This can be formed through segmentation of eigennetwork in $M$ sections of mental map called decision segment (Routenentscheidungsabschnitt), i.e. a sub-eigennetwork $\mathcal{S}_{mn} = (\mathcal{N}_{mn}, \mathcal{A}_{mn})$, $\mathcal{S}_{mn} \subseteq \mathcal{S}_n$. In this concept (see Figure 1), there are some alternatives to be chosen. The first alternative, route $b-e$, can be called the best route or the favorite path, which represents the path preferred by the traveler. The other alternatives are called the second best alternatives. In case there are two second best alternative routes: the first alternative is one with decision node and the second alternative is one without decision node, we can formulate a decision matrix as described below. The minimum requirement of a decision segment is the...
existence of one other alternative route. It is also possible that only one second best alternative is available at time $t$. If there are more than two other alternatives, the worst alternative(s) should be eliminated and then a new decision segment can be constructed recursively at the decision node $c$. The node $c$ should be expanded as node $b_1$, i.e. level one of the new decision segment starts at $c$ or $b_1$, includes new nodes $c_1$ and $d_1$, and ends at node $e$, etc. At decision node $c$, the traveler faces a decision to choose between route $c$-$d$ then $d$-$e$ and a direct route $c$-$e$. At node $d$ the traveler cannot change the route. Node $d$ is only a virtual node that allows a distinguishing feature of the alternatives. This node can also be located at the path $b$-$e$. Unavailable alternative(s) within this structure should be observed.

Let $P^r(t)$ be the set of paths connecting $r$ to $s$ at time $t$. It is assumed that the set of feasible paths in the network is explicitly enumerated and that a path’s origin $p_r$ and destination $p_s$ can be determined from the path’s identification. A trip of individual traveler $n$ or driver class $n$ is assumed to be following a path $p \in P^r(t)$ at all times. Figure 1 shows the structure of decision segment. Decision segment is a preliminary concept that needs to be developed further.

Choice of the alternatives. The decision making process within a decision segment is modeled as a decision problem that can be solved with FiPV algorithm. Two phases are considered, namely: phase 1 includes the possible alternatives at node $b$ and phase 2 consists of possible alternatives at node $c$. Let first discuss about FiPV.

Fuzzy Choice Function (FiPV)

There are fundamentally two approaches to modeling of individual choice: the popular one is based on utility theory or an absolute representation of preference leading to a numerical expression of preference intensity, the form in which preferences between outcomes are specified. Another approach is based on binary relations that encode pairwise preference, the form of specifying alternatives among which choices are to be made. Discrete choice models belong to the former, this paper is concerned with the latter.

Mathematical model. Fuzzy sets adopted in this paper is based on fuzzy individual choice in discrete decision space (cf. Orlovsky, 1978; Zimmermann, 1987) named FiPV. Modeling preference relation in simple terms means expressing preferences for all possible pairs $(x, y)$ of alternatives by providing answer to questions like: is alternative $x$ not inferior to alternative $y$? (Fodor et al., 1998). The most basic type of preference modeling is that of preference structure (De Baets & Fodor, 1997). Consider a set of alternative $A$ and suppose that a decision maker wants to judge them by pairwise comparison. Given two alternatives $x$ and $y$, the decision maker can act in one of the following ways: he clearly prefers $x$ to $y$; the $x$ and $y$ are indifferent to him; he is unable to compare $x$ and $y$. Three binary relations can be defined in $A$: the strict preference relation $P$, the indifference relation $I$, and the incomparability relation $J$. For any $(x, y)$ in $A^2$ we classify (cf. De Baets & Fodor, 1997; Fodor & Roubens, 1994):

$$
(x, y) \in P \Leftrightarrow \text{he prefers } x \text{ to } y,
(x, y) \in I \Leftrightarrow x \text{ to } y \text{ are indifferent to him},
(x, y) \in J \Leftrightarrow \text{he is unable to compare } x \text{ and } y.
$$

(5)
A binary relation $R$ with respect to each pair of alternatives $(x, y)$ of a given set $A$ is considered to be a weak preference relation. This and three binary relations corresponding to the given preference relation $R$ are defined as follows:

\[
\begin{align*}
    xRy & \iff x \text{ not worse than } y, \\
    xPy & \iff xRy \text{ and not } yRx, \\
    xLy & \iff xRy \text{ and } yRx, \\
    xJy & \iff \text{not } xRy \text{ and not } yRx.
\end{align*}
\]

Binary relations of alternatives $R(A^2)$ can be expressed by matrices with the properties: (a) the elements of the set $A$ is a representation of available choice alternatives; (b) matrix element $\mu_k(x, x_j)$ is a membership grade which represents pairwise comparison between alternative $A_i$ and alternative $A_j$ by the individuals subject to the specified attributes or decision criteria. If a fuzzy pairwise comparison matrix exists, the choice of the best alternative can be solved with a standard procedure proposed by Orlovsky (1978). Ranking of all alternatives and more practical choice procedure are provided by Ridwan (2000). In this paper we will not discuss about ranking/choice procedure, rather we discuss its structure in route choice process. By definition, FiPV is a choice based on $\mu_k(x, x_j)$ or

\[
FiPV \equiv \text{choice procedure based on } \mu_k(x, x_j)
\]

Application. Let $b, c \in \Psi^{rs}_{ap}(t)$ represent decision nodes within the smallest decision segment (see Figure 1). The decision making process can be modeled as a FiPV function. $FiPV_{ap}^{be}(t)$ represents a fuzzy decision function for route choice within a decision segment $b-e$ at time $t$ within interval $[t_r, t_s]_n$. The travel decisions from origin $r$ to destination $s$ are the sum of the decision of each segment. This can be expressed as

\[
FiPV_{ap}^{rs}(t_r, t_s)_n = \sum_{m=1}^{M} FiPV_{ap}^{be}(t)_n \quad \forall a, p, r, s, \quad \forall b, e \in S_{na}
\]

For simplicity, we omit the index $n$ of individual traveler. Since we wish to work with FiPV, equation (2) can be rewritten as follows

\[
x_a(t) = \sum_{ip} x_{ap}^{rs}(t) = \sum_{ip} f_p^{rs} FiPV_{ap}^{rs}(t) \quad \forall a
\]

where

\[
FiPV_{ap}^{rs}(t) = \begin{cases} 
1 & \text{if the link } a \text{ is part of path } p \\
0 & \text{otherwise}
\end{cases}
\]

The link $a$ is part of path $p$ means the traveler $n$ chooses path or sub-path on any link $a$ at time $t$ in the period $[t_r, t_s]_n$.

Decision matrix. The FiPV function is represented by its preference structure as shown above. If the traveler prefers to choose a route over a decision node $c$, then we can express the decision function as

\[
FiPV_{ap}^{be}(t) = [b \rightarrow c](t) + FiPV_{ap}^{ce}(t) \quad \forall a, p, b, c, e
\]
where \([b \rightarrow c](t)\) is a well-defined selected path whenever traveler \(n\) is entering the decision segment over the decision node \(b\) at time \(t\). The individual decision making process within the decision segment \(m\) (see Figure 1) can be divided in two decision phases as follows.

Phase 1: Possible alternative paths at decision node \(b\)

Alternative route 1: \(b \rightarrow e\)
Alternative route 2: \(b \rightarrow c \rightarrow e\)
Alternative route 3: \(b \rightarrow c \rightarrow d \rightarrow e\)
Alternative route 4: \(b \rightarrow d \rightarrow e\)


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Table 1 Decision matrix phase 1

In phase 1 the traveler has four possible alternatives. He has to compare all of these alternatives or he has to make 12 pairwise comparisons to decide which route to choose (see Table 1).

Phase 2: Possible alternative paths at decision node \(c\)

Alternative route 2: \(c \rightarrow e\)
Alternative route 3: \(c \rightarrow d \rightarrow e\)


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<tr>
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<td>(\mu'_{32})</td>
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Table 2 Decision matrix phase 2

In phase 2 the traveler has two possible alternatives for selection, he has to compare between the alternatives 2 and 3 only. This is the simplest form of fuzzy choice, if \(\mu'_{32} > \mu'_{23}\) the alternative 3 \(\succ\) (is more preferable than) the alternative 2 or alternative route 3 will be chosen; if \(\mu'_{23} > \mu'_{32}\) otherwise alternative route 2 will be chosen (see Table 2). FiPV provides a feedback element to guarantee the conditions that satisfy transitivity.

If the situation arises where the traveler considers searching other possible alternative(s) to switch the path anywhere, especially in a complex network, decision node \(c\) could be expanded to \(b_1\), recursively, as discussed before. A general formula for individual traveler choice behavior within a decision segment \(b-e\), \(FiPV_{ap}^{be}(t)\), can be derived from (9). Then we have

\[
FiPV_{ap}^{be}(t) = [b_i \rightarrow c_j](t) + FiPV_{ap}^{b_i,c_j}(t) + FiPV_{ap}^{c_j,e}(t) \quad \forall a, p, b, c, e
\]

where \(b_i\) is the parent node of \(b_{i+1}\) (former \(c_i\) ) node. In case the traveler decides not to select the path over the node \(c_{i+1}\), then \(FiPV_{ap}^{c_{i+1},e}(t) = 0\). Consequently, either path \(c_\varphi e\) or \(c_\varphi d-e\) would be chosen.
Inductive Reasoning

Modern psychologists are in reasonable agreement that in situations that are complicated or ill-defined, humans use characteristic and predictable methods of reasoning. These methods are not deductive, but inductive. Arthur (1994) introduced El-Farol problem to illustrate inductive reasoning. Similarly, Nakayama and Kitamura (2000) argued that travelers’ reasoning is not a deductive reasoning in which abstract and normative rules are applied. Thus drivers when faced with problems seek regularities and build hypotheses, verify them as rules, and apply them; travelers reason and learn inductively. They show an example of induction in route choice behavior as follows. Suppose a driver has experienced repeatedly that a route was not congested the day after it had been heavily congested. This driver would then start anticipating, after experiencing congestion on the route one day, that the route will be uncongested the next day. In this case experiences are generalized and a hypothesis is formed, which is then stored as a piece of knowledge and applied when a similar situation arises again. This cognitive process is induction. We try to incorporate inductive reasoning in our model, namely in the formation of inductive rule base.

Uncertainty in anticipated travel time. Let first assume that travel time over path \( p \) is approximately \( t_u \) minutes if the system performs a normal traffic and approximately between more than \( t_u \) minutes and more than \( t_c \) hours if congested. Traveler \( n \) anticipates these travel times with a confidence level of \( \mu_u \) and \( \mu_c \) respectively.

\[ \mu \text{ (level of confidence)} \]

\[ \mu_u \]

\[ \mu_c \]

\[ t_u \text{ min} \]

\[ t_c \text{ hr} \]

Figure 2 Uncertainty of travel time

The global anticipated travel time or cost can be formulated as

\[ c_p^b(t) = \bigcup \mu_i \cdot c_{pi}^b(t) \] (13)

Henn (2000) used this formula to describe paths predicted cost of the network to study traffic dispersion in a macroscopic traffic assignment model at the presence of VMS.

As shown in Figure 2, \( c_{pi}^b(t) = \mu_u \cdot t_u^b(t) \cup \mu_c \cdot t_c^b(t) \) represents the anticipated travel time from node \( b \) to node \( e \). Example: path \( b-e \) can be traveled at normal condition (uncongested) ca. 45 minutes and at congestion ca. 2 hours. Suppose a traveler \( n \) will compare two travel situation, at 7:30 and 8:30. The confidence level that at 7:30 no congestion to be happened is 0.95, but it could be congested by the level of confidence of 0.20. At 08:30 the route \( b-e \) will be jammed with a confidence level of 0.80, but it could be normal with a confidence of 0.40. This can be expressed as:
These uncertainties cannot be properly quantified well, however, a comparison between both functions is quite easy. It is not hard to conclude that travel at 7:30 is better than at 8:30. The problem will be more complicated if we have to compare travel times of two different paths as shown on the following comparison. Suppose we want to evaluate travel possibilities at 7:30. Travel time of a direct path from node $b$ to $e$ is as stated above. Travel time from node $b$ to $e$ through node $d$ (see Figure 1) is 30 minutes with a confidence level 0.60 and 90 minutes with a confidence level 0.50. This can be written as

$$ c_{be}^{bd}(7:30) = 0.95 \cdot (45 \text{ min}) \cup 0.20 \cdot (120 \text{ min}) $$

$$ c_{be}^{bde}(7:30) = 0.60 \cdot (30 \text{ min}) \cup 0.50 \cdot (90 \text{ min}) $$

In a natural way, the process of comparisons occurs in a fuzzy environment that also depends on the traveler’s experience. Thus the values of $\mu_u$ and $\mu_c$ must be obtained from the continuously observations of individual experiences, since these values are always changing in accordance with changes in the traffic conditions. However, it is not easy to observe these numerical values precisely. We leave this, even though, behind pairwise conception.

**Information processing.** It is reasonable to expect that travelers’ perceptions would change with time. Therefore, it is important to model the process by which the travelers update their perception. In pre-trip, the travelers combine the experience from the last day with the actual information collected before entering the trip. Travelers use the pre-trip information and make a comparison with the real-time information they are receiving during the en route. In post-trip, travelers evaluate the quality of information today and use it for tomorrow’s trip as experience. All of these process can be recorded in the decision matrices, since FiPV structure includes time and space dimensions. Experienced travelers might be assigned a level of knowledge at which they do not need information anymore, but merely a high quality real-time information. Inexperienced travelers, otherwise, need anyway to be well informed.

Inexperienced travelers use a map to help themselves finding the next possible decision node. Experienced travelers, on the other hand, use their memory (knowledge). Travelers generally experiment the trips, anticipate the traffic condition incl. travel times. Thus the values of $\mu_u$ and $\mu_c$ depend on the level of experience of the traveler. The role of past experience on present decision has been intensively studied in psychology. A recent research about script-based choice (Gärling et al., 2001) may help us to understand such phenomena.

The general formula (12) can be used as search path algorithm for a travel from origin $r$ to destination $s$, in which the individual criteria are not generalized or simplified like the assumptions of the most used probability choice models. However, this model could be also combined with the usual optimization criteria (based on disaggregate representation) to get better results, since observed FiPV data are not always available.

In case of (11)-(12), origin $r$ is defined as the start decision or node $b$ and destination $r$ as the end decision or node $e$ of the parent decision segment. Whenever the individual preference data is not available or not completed, we can use the standard criteria e.g. $\sigma_o^u(t) > \sigma_o^h(t)$ to approximate the elements of the pairwise comparisons matrix, $\mu_p(x_i, x_j)$. All possible selected paths through the networks can be modeled within the FiPV structure. This decision structure represents a bounded rationality model of human thought and choice process, especially the limits of human cognitive capacity for discovering alternatives, computing their
consequences, and making comparisons among them. It should be noted that decision segment \( S_{mn} \) is not a physical sub-network, but mere a mental map of the sub-network, i.e. a subset of eigennetwork \( S_n = (N_n, A_n) \), \( S_m \subset S \). Thus \( S_{mn} = (N_{mn}, A_{mn}) \mid S_m \subset S \). Travelers can only choose from alternatives of which they are aware, consequently information availability affects the construction of eigennetwork \( S_n \), therefore induces decision segment \( S_{mn} \) as well; aspects that we need to pursue. In modeling real world, we have to consider the decision nodes in their natural form, with intersections etc., not simply as a point.

**Inductive rule-based approach.** In a transportation system, travelers experiment with alternative routes, listen to traffic information on radios etc., and compare experiences with other travelers. Based on this evaluation, travelers settle on routes which are most preferred. This process can be captured in the FiPV model as inductive rule-based procedure, which can be characterized by

\[
\text{If } \kappa \text{ then } \mu_{xy} >>> \text{ and } \mu_{yx} <<< \\
\text{If } \gamma \text{ then } \mu_{xy} <<< \text{ and } \mu_{yx} >>>
\]

where \( \kappa \) and \( \gamma \) represent performance changes on the system or different circumstances of the traveler. The effects of \( \kappa \) and \( \gamma \) are reciprocal. If \( \kappa \) implies the value of \( x \succ y \) is increased, i.e. \( \mu_{xy} \) becomes greater (>>>); consequently the value of \( y \succ x \) will be decreased, i.e. \( \mu_{yx} \) is growing less (<<<), then \( \gamma \) causes the reverse. The \( \mu_{xy} \) and \( \mu_{yx} \) values formation is an inductive process. These form a new decision matrix \( \mu_R(x_i,x_j) \) in which \( x_i = x \) and \( x_j = y \). The new decision matrix may cause a new choice and the formation of the matrix elements \( \mu_R(x_i,x_j) \) continues.

**Example**

The following example will demonstrate the FiPV \( \mu_R(x_i,x_j) \)-choice function (7) that can be manipulated at the laboratory or compared with the real decision in the transportation system.

We present examples to show the different between traveler who cannot maximize his behavior and traveler who is able to maximize his behavior. The maximizing behavior is the behavior of individuals who optimize their travel criteria such as shortest path or least time. Their behavior can be predicted approximately with shortest path algorithm, minimizing impedance or disutility (travel time, cost, fuzzy cost, etc.) or maximizing utility function. However, the individual perceptions of perceived travel time are varied. In contrast, the non-perfect-maximizing behavior cannot be predicted from a general formulation like that. This kind of behavior must be observed empirically, and the behavior pattern subsequently studied.

We show decision feedback as preference formation and inductive rule-based process as context dependence in the following example (see also Figure 1 and Table 1 & Table 2). Given crisp travel time data (no information about distribution, the travelers only know about minimal and maximal travel times in uncongested network) as follows:

- alternative 1: 18 minutes (normal), 40 minutes (congested)
- alternative 2: 20 minutes (normal), 35 minutes (congested)
- alternative 3: 25 minutes (normal), 30 minutes (congested)
- alternative 4: 16 minutes (normal), 40 minutes (congested)
Non-perfect-maximizing behavior

At decision point $b$

<table>
<thead>
<tr>
<th>Favorite route</th>
<th>Stated response</th>
<th>Preference formation (feedback)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1  2  3  4</td>
<td>1  2  3  4</td>
</tr>
<tr>
<td>1</td>
<td>1,00 0,90 0,80 0,70</td>
<td>1,00 0,90 0,80 0,80</td>
</tr>
<tr>
<td>2</td>
<td>0,60 1,00 0,30 0,40</td>
<td>0,60 1,00 0,30 0,40</td>
</tr>
<tr>
<td>3</td>
<td>0,70 0,50 1,00 0,20</td>
<td>0,70 0,50 1,00 0,20</td>
</tr>
<tr>
<td>4</td>
<td>0,80 0,10 0,40 1,00</td>
<td>0,80 0,10 0,40 1,00</td>
</tr>
</tbody>
</table>

--> intransitive

Travel information:
Congestion on path $b$-$e$

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,00 0,60 0,80 0,70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0,80 1,00 0,30 0,40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0,70 0,50 1,00 0,20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0,80 0,10 0,40 1,00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\mu_{ND}$ 0,80 0,80 0,80 0,70

--> no decision

At decision point $c$

The traveler changes his mind

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,00 0,60</td>
<td></td>
</tr>
<tr>
<td>0,70 1,00</td>
<td></td>
</tr>
</tbody>
</table>

$\mu_{ND}$ 0,90 1,00

--> to be chosen: alternative 3 or path $c$-$d$-$e$

If the traveler has a behavior that does not satisfy maximizing travel objectives (e.g. because of unfamiliarity, lexicographic choice, non-compensatory, etc.), the researcher may observe the behavior through fuzzy preference values $\mu_R(x_i,x_j)$ in a decision situation with respect to which the traveler has made pairwise comparisons between available alternatives $i$ and $j$ in his decision segment. The values are different from those of traveler who can maximize his travel objectives. Suppose we got some values from a real observation like the first matrix at the left that is obviously intransitive. Assume that the traveler will choose the alternative 1 or path $b$-$e$, but this matrix is not easy to interpret. We suggest therefore a feedback to form a transitive condition like the matrix at the right. The new FiPV matrix gives a better interpretation about the traveler’s choice. Yet, when the pre-trip information announced that congestion occurs on path $b$-$e$, the traveler will start to revise his preference, perhaps first with the result: no decision. After reevaluation he chooses alternative 2 or path $b$-$c$-$e$. En route information or other condition like incident can then affect the traveler to make another decision while he is on the route segment $b$-$c$, e.g. to choose path $c$-$d$-$e$ as shown above.

Pure rational utility-maximizer without information

We will show first a simple problem where traveler as a utility-maximizer has to optimize his travel objectives in uncongested network and then in congested network without information. We see the fact that in the normal (uncongested) network the chosen path is the least time.
But we also see that in congested route, the traveler behaves other than normal. The rational traveler tries to seek a better alternative by guessing that the other route may not be over capacity. The traveler do not know about the traffic condition precisely, he only guesses and think about his habit or his current decision that could be reasonable to be revised, he want to make a new comparison of possible alternatives. The process continues during the traffic jam. If he finally finds the other route better than his current route, he will change the route. The updating process consists of mere guessing. We will see the reality how in the congestion the actual best route (alternative 3 route b-c-d-e: 30 minutes) could not be found by the traveler. Conventional shortest path methods if applied here will result a fundamental error.

Rational utility-maximizer with information

The problem is like that of the first example (non-perfect maximizing behavior), except the fact that the traveler can or will exact follow the maximum criterion designed/assumed by the researcher. Here the best route can be found by the traveler with traffic information. In other paper we discuss a numerical example about risk behavior of the traveler in similar context.

Both travelers’ behaviors, perfect maximizing and non-perfect-maximizing, can be modeled in a one form FiPV matrix. The $\mu_{R}(x_i, x_j)$ values of the maximizing behavior correspond directly to the impedance/disutility, i.e. identical with fuzzy cost (Henn, 2002; 2000), but these fuzzy preference values of the non-perfect-maximizing behavior do not.

**Conclusions**

The main purpose of the paper is to demonstrate that route choice can be modeled within FiPV framework. FiPV (fuzzy traveler preferences) model is a choice function based on fuzzy preference relations. This modeling approach has been developed by the author at the RWTH Aachen to improve travel demand analysis. The proposed methodology described in this paper needs to be validated against empirical observations. This is a task for future research. Although our model has its own character, practical comparison of the proposed model with
probabilistic route choice model (e.g. Bekhor et al., 2002; or Cascetta et al., 2002) is quite possible. From theoretical viewpoint, however, it’s obvious that route choice in probabilistic model is equivalent to the portion of our representation for perfect maximizing behavior as normally assumed, the other kind of behavior remains unable to be modeled by the conventional model. In the real world, we cannot distinguish between non-maximizer and utility-maximizer travelers.

Individuals will try to maximize their travel objectives. However, this aim is a long process of learning and adjustment that may not be achieved while the individuals are traveling. It is argued that travelers do not or cannot always follow the perfect utility maximization principle. Therefore we formulate a model that also considers the travelers with non-perfect-maximizing behavior. We adopt some ideas of alternatives perception and mental map that can reduce hundreds of possible alternatives to a limited number of real potentially available path alternatives that are actually faced by travelers. The example shows the model structure and the behavior distinction between two types of travelers. The fact that travelers do not always follow the shortest path or least travel time/cost can be explained in terms of FiPV model. The values of fuzzy preference relations \( \mu_{ij}(x_i, x_j) \) reflect the individual interpretation of the choice situation that absolutely independent from general principle of optimization or maximization. However, these can also handle situation in which the travelers maximize their travel objectives. In contrast, utility as overall criterion cannot always represent subjective criteria that may also depend on decision context.

Inductive rule-based approach (equation 14) introduced in this paper helps the researcher to obtain dynamic numerical value of the membership function \( \mu_{ij}(x_i, x_j) \) that represents traveler’s perception update as a result of his adaptation process in making travel decision. The structure of inductive rule base is flexible so that dynamic reactions of individual traveler to continual change on transportation system performance and other factors influencing his travel decision can, therefore, be modeled realistically. The example shows that as rational utility-maximizer, traveler seems to have no possibilities in choosing the route. In other words, route choice models mainly developed on the basis of the assumption as if travelers are utility-maximizers cannot really describe behavior of all individuals under any circumstances. Parallel to the results in decision theory, economics and psychology, it appears that time has come to ask about the plausibility of alternative approach to extend the traditional model.

References


TRB 2003 Annual Meeting CD-ROM    Paper revised from original submittal.


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