TEST OF A BEHAVIORAL THEORY OF MULTI-LANE TRAFFIC FLOW: QUEUE AND QUEUE DISCHARGE FLOWS

by

James H. Banks
Civil and Environmental Engineering Department
San Diego State University
San Diego, CA 92182-1324
Phone: (619) 594-7051
FAX: (619) 594-8078
E-mail: banks@mail.sdsu.edu

and

Mohammad R. Amin
Civil and Environmental Engineering Department
San Diego State University
San Diego, CA 92182-1324
Phone: (619) 594-7131
FAX: (619) 594-8078
E-mail: mamin@rohan.sdsu.edu

Word count: 5,267 + 7 figures x 250 = 7,017

Submitted to the Transportation Research Board for publication and presentation
Revised, 11-12-02

ABSTRACT

Daganzo (*1*,*2*) has recently proposed a behavioral theory of multi-lane traffic flow. This theory is the basis for several predictions related to flow phenomena, including several about the way that the relative speeds and flows in different lanes change in transitions to and from congested flow. In particular, the theory asserts that the more aggressive drivers (referred to as “rabbits”) always behave so as to maximize their speed. In congested flow, speeds are assumed to be nearly equal for all the lanes, and rabbits distribute themselves across the lanes so as to maintain this equality of speed. In acceleration downstream from queues, the equality of speed among the lanes breaks down once the free flow speed of the slowest lane is reached, and the rabbits segregate themselves in the fastest lane. This redistribution of flow is expected to lead to two distinct flow states in queue discharge, referred to as “capacity flow” and “discharge flow.” Because the transition from capacity flow to discharge flow may lead to different overall flows and densities in the two states, a wave marking the transition between them may move either upstream or downstream. Automatically-collected flow, occupancy and speed data from freeways in the San Diego and Toronto areas were used to test these features of the theory at merge bottlenecks and in queue discharge following the removal of incidents. It was found that there are often speed differences among the lanes even in congested flow and that sudden redistribution of flow across the lanes can occur without speed equalization. These findings imply that Daganzo’s behavioral logic is oversimplified and that something more than just maximization of speed by aggressive drivers is involved in the distribution of flow across the lanes.

Daganzo has recently proposed a behavioral theory of muli-lane traffic flow (*1*,*2*). In the idealized form developed so far, this theory assumes two types of drivers, aggressive (referred to as *rabbits*) and timid (referred to as *slugs*), and two lane groups: *shoulder lanes* and *passing lanes*. In free flow, rabbits travel faster than slugs and the two groups are segregated in *two-pipe* flow, with the slugs all in the shoulder lane and the rabbits all in the passing lane. In high-volume uncongested flow, rabbits follow one another with very small headways so long as they are able to pass, and such drivers are referred to as being *motivated* because the very small headways are held to be motivated by the desire to pass slower vehicles. Whenever an event occurs that reduces speed in the passing lane to (or below) v_f, the free-flow speed in the shoulder lane, the rabbits change lanes to equalize speeds, lose their "motivation," and increase their headways. This results in *one-pipe* flow. Because of the difference in headways, maximum uncongested flow rates in the passing lane exceed its queue discharge rate.

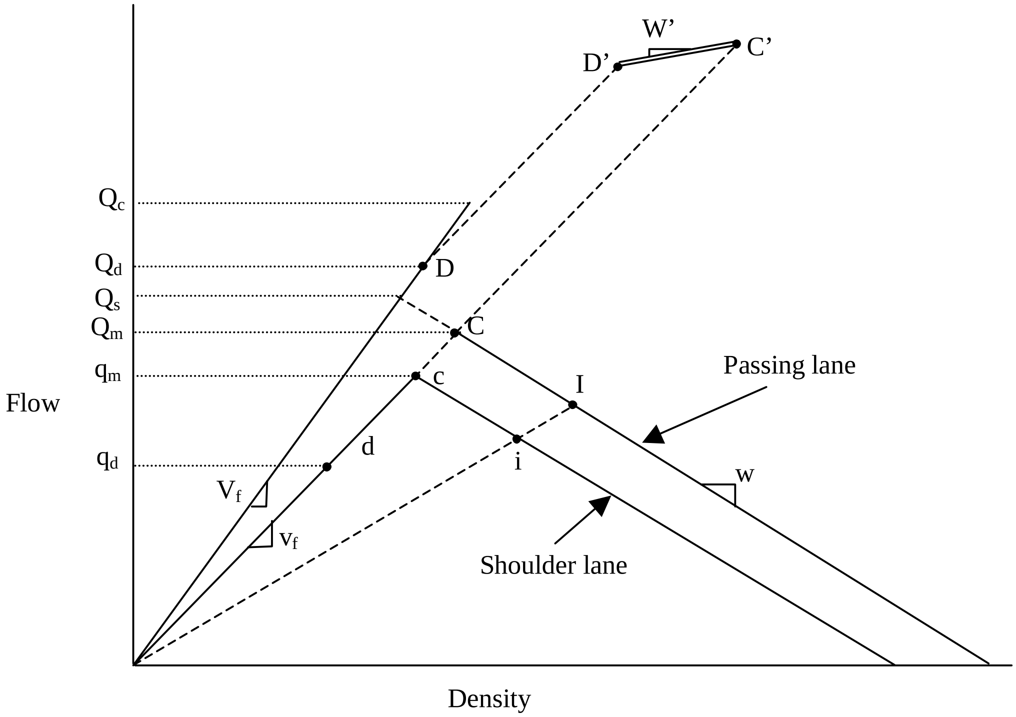
Daganzo provides solutions for several cases involving transitions from two-pipe to one-pipe flow and vice versa, and predicts a number of potentially observable phenomena and the circumstances under which they should occur. Among these are *semi-congested* states (in which there are queues in some lanes but not others), so-called *fast wave speeds*, and distinct *discharge* and *capacity* states in flow downstream from queues. This paper reports on efforts carried out at San Diego State University to test aspects of the theory related to flows in queues and queue discharge, in which the characteristics of discharges from both fixed merge bottlenecks and incident queues were compared with some of the predictions of the theory. Additional research to test the theory was carried out at the University of California at Berkeley but is not discussed here.

PREDICTIONS RELATED TO QUEUE DISCHARGE FLOWS

Figure 1 is a flow-density diagram that illustrates certain features of Daganzo's theory, as they relate to conditions in and downstream of queues. The theory, as illustrated by the diagram, assumes constant free flow speeds V_f on the passing lane and v_f on the shoulder lane; identical wave speeds w for both lanes in congested flow, and hence identical slopes for the congested branches, but with that for the passing lane above that for the shoulder lane; a so-called reverse lambda shape for the passing lane diagram, with maximum flows occurring in free flow; and a triangular shape for the shoulder lane diagram. In general, capital letters refer to features related to the passing lane and lower case letters to those related to the shoulder lane. Other important features include *critical flow* Q_c, the maximum flow on the passing lane in free flow; *saturation flow* Q_s, that is, flow at the projected intersection of the right branch of the diagram with the V_f-ray; Q_m, the *capacity flow* on the passing lane in the one-pipe state; and q_m, capacity flow on the shoulder lane.

A fundamental feature of the theory is that rabbits distribute themselves in the lanes so as to maximize speed, but slugs always remain in the shoulder lane. In acceleration out of one-pipe queues, rabbits will use all lanes so long as speed is equalized across the lanes, but will switch to the faster lane (or lane group) when it is not. Daganzo assumes that in the queue, speed will be the same in both lanes, and less than free flow speed in either of them; this condition is illustrated on the diagram by points *I* and *i* (representing rabbits and slugs respectively) and the dashed line connecting them to the origin, whose slope represents the speed in the queue. As vehicles accelerate

FIGURE 1 Assumed Flow-Density Diagram



downstream of the queue, the points representing the two driver populations will move up until they reach points *C* and *c*. At this time, speed in both lanes is equal to v_f, the normal free-flow speed in the shoulder lane. This condition Daganzo refers to as the *capacity state*. The rabbits in the passing lane will continue to accelerate, however, and as soon as their speed exceeds v_f, rabbits in the shoulder lane will switch to the passing lane. Eventually, the speed of the rabbits returns to V_f, and the flow is represented by points *D* and *d*, referred to as the *discharge state*. Daganzo assumes that Q_d, the rabbit flow in the discharge state, will normally exceed Q_m but be less than Q_c.

The transition from the capacity state to the discharge state will normally involve changes in total flow and density as well as in the relative use of the lanes. Density is assumed to decline in the transition, but flow may either increase or decrease. The rabbit discharge flow Q_d is assumed to be a characteristic of the flow process, but the slug discharge flow is a function of the relative number of rabbits and slugs in the traffic stream. If the fraction of slugs is high, the combined flow will be large, and may exceed the combined flow in the capacity state; if not, it will be less. Flows and densities in the two states may be added to give points *C’* and *D’*. If the slope of a line connecting these points (*W’* on the diagram) is positive, the transition between them will move downstream; if it is negative the transition moves upstream.

These considerations lead to the conclusion that (according to the theory) an observer downstream of a queue (beyond the zone of acceleration) might see two distinct flow states in the queue discharge. The first of these to be observed following the beginning of the discharge (either the activation of a fixed bottleneck or the removal of an incident) would be the discharge state. This hypothetical state is characterized by speeds and speed differentials between lanes similar to those in free flow, by a lane use distribution similar to that in free flow, but with flow in the passing lane somewhat less than the maximum free flow value. Following this, the observer might see a transition to the capacity state, characterized by similar speeds for both lanes, a more equal distribution of flow than in free flow or the discharge state, and (because it is required for the transition to move downstream) an increase in total flow. In the event that flow did not increase in the transition, a downstream observer would see only the discharge state.

DATA AND METHODOLOGY

The theory was tested by comparing the features outlined above with selected characteristics of flow in the vicinity of merge bottlenecks and incidents. Data from five merge bottlenecks and six incidents were analyzed. Four of the bottlenecks were in the San Diego area and the fifth was on the Queen Elizabeth Way (QEW) in the Toronto area; all six of the incidents were in the San Diego area.

Merge bottleneck sites were as follows:

- Site 1. Southbound Interstate 5 downstream from Manchester Avenue, morning peak
- Site 2. Westbound Interstate 8 downstream from Fletcher Parkway, morning peak
- Site 3. Eastbound QEW, downstream from Cawthra Road, morning peak
- Site 4. Southbound Interstate 805, downstream from Nobel Drive, evening peak
- Site 5. Northbound Interstate 5, downstream from Via de la Valle, evening peak

Results for the morning and evening peak bottlenecks were similar; the three morning peak cases will be discussed in detail because they illustrate the range of the results and allow comparison of the San Diego and Toronto data.

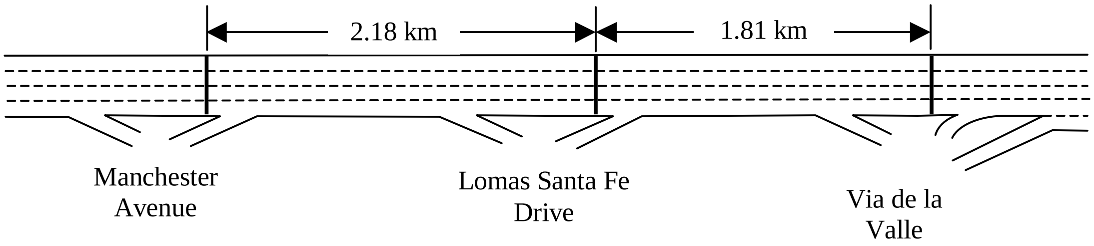
Figure 2 shows lane configurations for the morning peak bottlenecks. Time series of speeds at these locations were compared with similar data from locations immediately downstream to confirm that local congestion was not the result of queues growing into these sections from downstream. These comparisons established that each location was an active bottleneck for at least part of the morning peak. The exact location of the bottleneck within each section was not always obvious and will be discussed in more detail later. Site 2 was typically an active bottleneck for only the earlier part of the peak, with queues from downstream eventually growing into the section.

Incidents were identified by monitoring the California Highway Patrol computer aided dispatch system log (*3*) for incidents occurring in the San Diego area. Six incidents, all accidents that had a substantial impact on traffic flow, were studied in detail. Locations and times of these incidents were as follows:

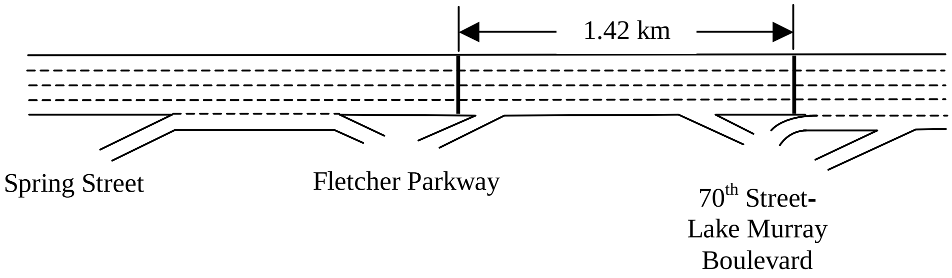
- Westbound Interstate 8 just west of Fletcher Parkway, 1:01 p.m., November 29, 2001
- Westbound Interstate 8 between College Avenue and Waring Road, 7:26 a.m., December 3, 2001
- Southbound Interstate 15 between Adams Avenue and El Cajon Boulevard, 2:17 p.m., May 9, 2002
- Eastbound Interstate 8, just upstream of Fletcher Parkway, 4:22 p.m., June 3, 2002
- Southbound Interstate 5 between Manchester Avenue and Lomas Santa Fe Drive, 2:35 p.m., July 2, 2002

FIGURE 2 Lane Configurations at Morning Peak Merge Bottleneck Study Sites

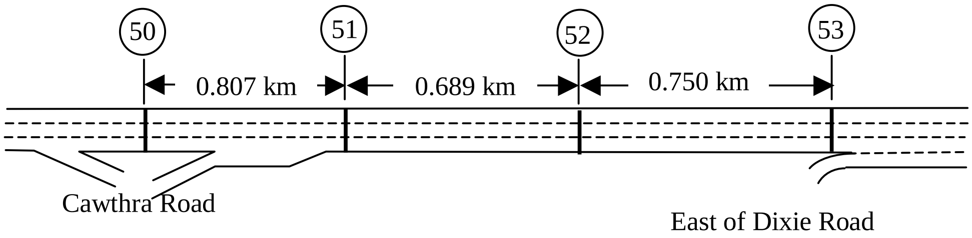
Interstate 5 Southbound



Interstate 8 Westbound



Queen Elizabeth Way Eastbound



Not to Scale

- Eastbound State Route 78, between Twin Oaks Valley Road and Barham Drive, 3:30 p.m., July 5, 2002

Preliminary investigation was carried out for a number of other incidents, but they were rejected because data were missing or incident queues appeared to have dissipated before the incidents were completely removed.

One challenge in testing Daganzo's theory was that it was not possible to find study sites that matched all its assumptions. First, the theory is developed in detail only for freeways with two lanes in the direction of flow, yet all the sites (both for merges and incidents) involved three or more lanes. The theory may be adapted to sections with more than two lanes by either grouping some of the lanes together or by assuming driver types intermediate between rabbits and slugs. In either case, the most aggressive drivers should be found in the lane closest to the freeway median (referred to as the median lane) and the least aggressive in the shoulder lane. Consequently, drivers in the median lane may be expected to resemble Daganzo's rabbits, and those in the shoulder lane to resemble slugs, regardless of what happens in the intermediate lanes. With that in mind, the analysis here will focus on comparing speeds and flows in the median lane with averages for the freeway as a whole.

A second mismatch between the theory and the characteristics of available sites is that detectors in the San Diego system are located immediately upstream of freeway entrances. Consequently, there were flows onto or off of the freeway between the downstream fronts of the queues and the measurement sites. Such flows could obviously alter the distributions of flow and speeds across the lanes, so that speed and flow distributions at different locations might not be comparable. Because of this problem (and the possibility that data from individual detectors may be biased) the primary evidence for changes in flow states was taken to be changes over time in the lane-by-lane speed or flow distributions at individual locations rather than comparisons between locations.

Finally, Daganzo's discussion of incident clearance flow assumes no interference by bottlenecks either upstream or downstream from the site of the incident. In practice, at least some of the incident queue discharges were affected by bottlenecks. The presence of these bottlenecks limited the amount of time for which pure incident recovery flow could be observed.

Data available at the San Diego sites consisted of 30-second volumes and occupancies for individual lanes produced by single-loop detectors. Speeds for the San Diego sites were estimated from volumes and occupancies by

$$\hat{u} = \frac{q\bar{L}}{\Omega} \tag{1}$$

where $\hat{u}$ = estimated speed
q = flow
$\bar{L}$ = average effective vehicle length
Ω = occupancy, as a dimensionless ratio

The average effective vehicle length assumed was that used by the California Department of Transportation for speed calculations for the San Diego system (24.75 ft or 7.5 m), and was the same for all lanes. Because of this practice, speeds in the outer lanes, which have a higher percentage of large vehicles, tend to be underestimated. Data at the QEW site were produced by double-loop detectors and consisted of 20-second speeds, volumes, and occupancies for individual lanes. In this case, speeds were measured directly. At each merge bottleneck site, data from between 7 and 9 different days were analyzed.

Daganzo's theory predicts that distinct capacity and discharge flow states will be observed downstream from freeway queues. These hypothetical states are characterized by different distributions of flow and speed across the lanes, as well as by differences in average speed and average flow. To test these predictions, flow and speed distributions were characterized in terms of the ratio of flow or speed in the median lane to the average flow or speed for all lanes. These ratios will be referred to as the *flow ratio* and *speed ratio* respectively, and are defined as

$$r_q = \frac{nq_1}{\sum_{i=1}^{n} q_i} \tag{2}$$

and

$$r_u = \frac{nu_1}{\sum_{i=1}^{n} u_i} \tag{3}$$

where r_q = flow ratio
r_u = speed ratio
q_i = flow in lane *i*, with lanes numbered outward from the median
u_i = speed in lane *i*
n = number of lanes

A major concern in the analysis was to identify changes in the time series of the speed and flow ratios. In addition, changes in the time series of speed and flow, both for individual lanes and for the freeway as a whole, were used to locate active bottlenecks and to identify the times of events such as transition to and from congestion, incident removal, and transition from incident queue discharge back to free flow. Re-scaled cumulative curves (*4*) and event-based averaging (*5*) were used as data smoothing techniques to identify and quantify these changes. In the case of re-scaled cumulative flow curves, the average slope of the cumulative curve represents the average rate of the time series; consequently, changes in the average of the underlying time series are signaled by changes in the slope of the cumulative curve. In the case of the flow and speed ratios, the re-scaled cumulative functions were calculated as

$$R_q(T) = \sum_{t=0}^{T-1} \left(r_{qt} - 1 \right)$$

and

$$R_u(T) = \sum_{t=0}^{T-1} \left(r_{ut} - 1 \right) \tag{4}$$

where $R_q(T)$ = re-scaled cumulative flow ratio prior to time T
$R_u(T)$ = re-scaled cumulative speed ratio prior to time T
r_{qt} = flow ratio for time period t

r_{ut} = speed ratio for time period t

The re-scaling is accomplished by accumulating the terms $r_{qt} - 1$ and $r_{ut} - 1$ rather than r_{qt} and r_{ut}, thus rotating the cumulative curve so that a horizontal line indicates that the flow or speed in the median lane is equal to the average for all the lanes.

When stated in terms of re-scaled cumulative flow and speed ratios, Daganzo's theory predicts that in free flow there should be positive slopes for both R_q and R_u, since median lane speeds and flows will exceed the averages for all lanes. Changes in these slopes over time are possible if the composition of traffic changes, but abrupt changes are unlikely.

In the queue and the zone of acceleration immediately downstream, there should be abrupt decreases in the slopes of both R_q and R_u at the transition to congested flow. If all speeds are measured accurately, R_u should become horizontal, since speeds in all lanes are hypothesized to be equal, although the slope of R_q should continue to be positive. Biases in speed or flow measurements might result in either positive or negative slopes for either R_q or R_u, however. The transition back to free flow when the queue clears should result in abrupt increases in both slopes.

Downstream of the zone of acceleration, the initial state is hypothesized to be the discharge state. Since speeds in this state are the same as in free flow and relative flows depend on the composition of traffic, there should be no change in either slope at the onset of congestion upstream; however, the theory does predict a decrease in average flow. Later, if there is a transition to the capacity state, both slopes should decrease and average flow should increase. When the queue upstream clears, there should be either an abrupt increase in both slopes (if flow is in the capacity state) or no change (if it is in the discharge state).

In all cases, changes in the slopes of R_q and R_u should coincide. It is a fundamental behavioral assumption of the theory that rabbits change lanes to maximize their speeds. Consequently, in the transition to congested flow, rabbits exit the median lane *because* speeds have equalized across the lanes, and in the transition to the discharge state, they move back into the median lane *because* speeds are no longer equal in all lanes.

RESULTS

Figures 3 and 4 show plots of re-scaled cumulative speed and flow ratios for representative days for locations at the downstream ends of the merge bottleneck study sections. These may be compared with Figures 5 and 6, which show similar plots for locations that are definitely upstream of the points of flow breakdown, as determined by comparing time series of speeds for different locations. In order to render the data comparable, all ratios were calculated for one minute intervals (the smallest common time interval for the San Diego and Toronto data bases) and the time scale is measured in hours after data collection began (5:30 a.m. in San Diego and 6:00 a.m. in Toronto).

FIGURE 3 Re-scaled Cumulative Speed Ratios for Downstream Locations at Merge Bottleneck Study Sites

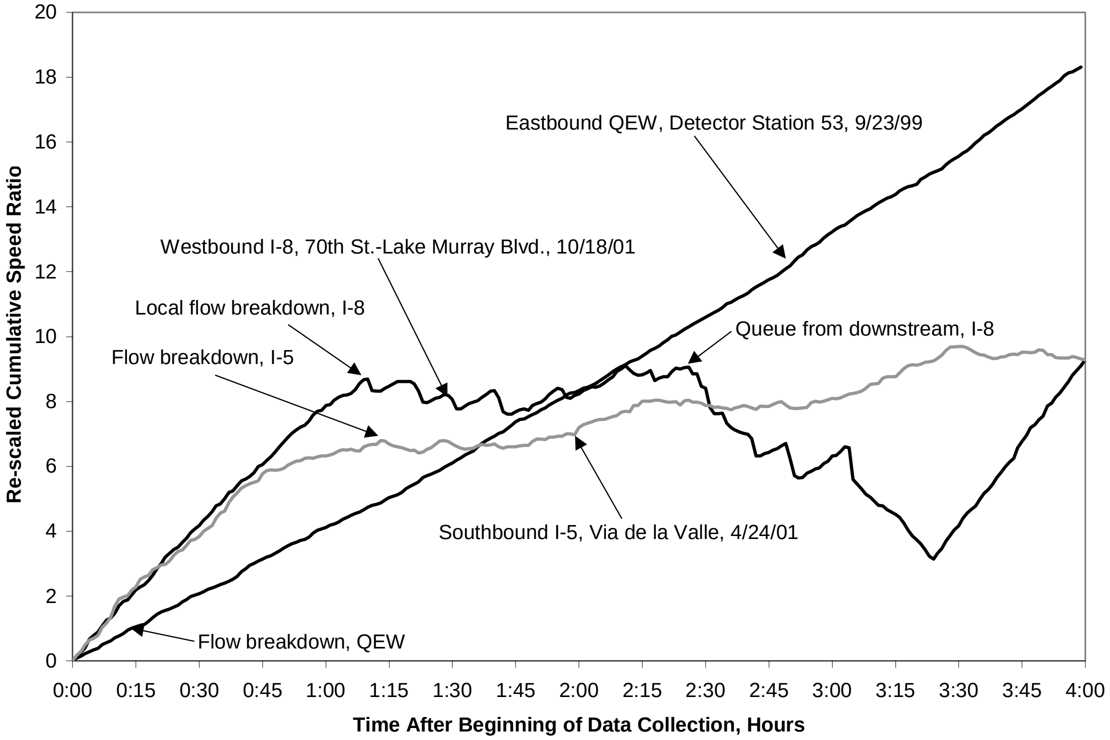


FIGURE 4 Re-scaled Cumulative Flow Ratios for Downstream Locations at Merge Bottleneck Study Sites

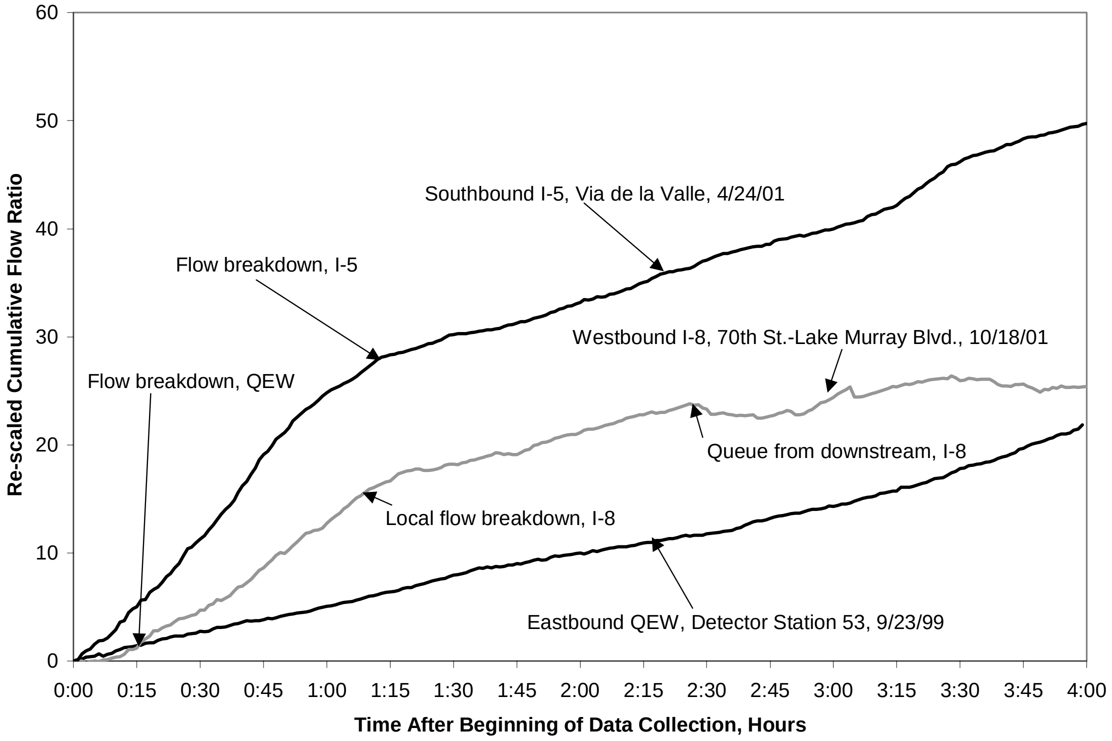


FIGURE 5 Re-scaled Cumulative Speed Ratios for Locations Upstream of the Point of Flow Breakdown

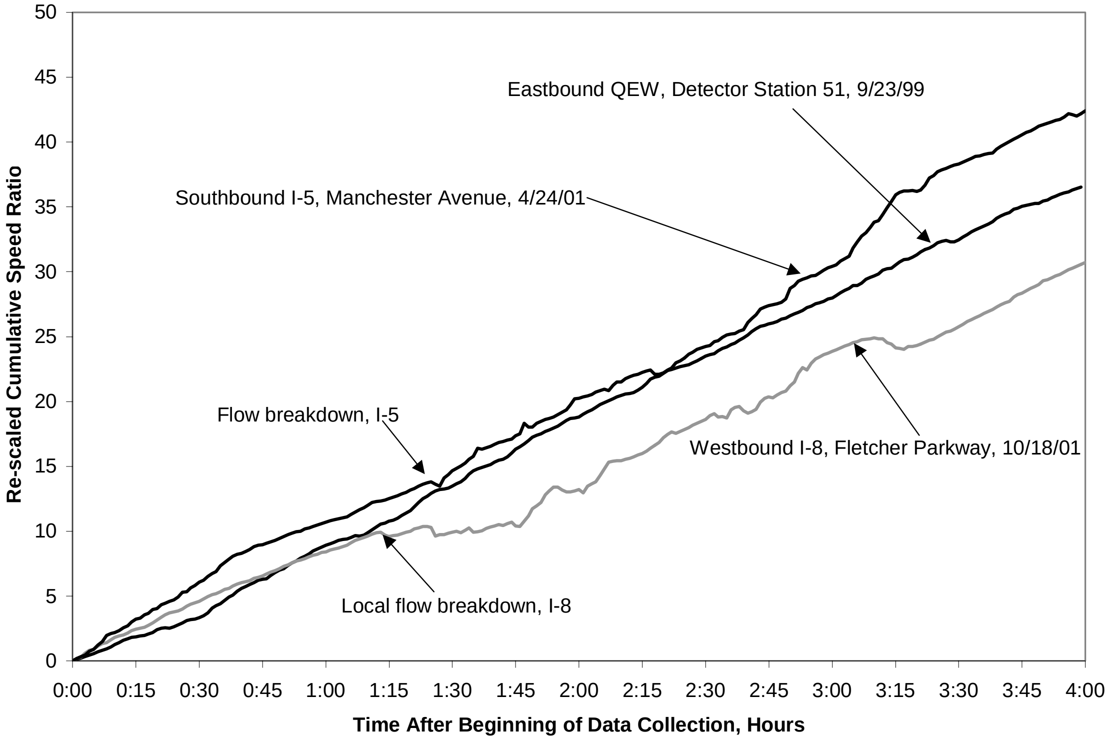
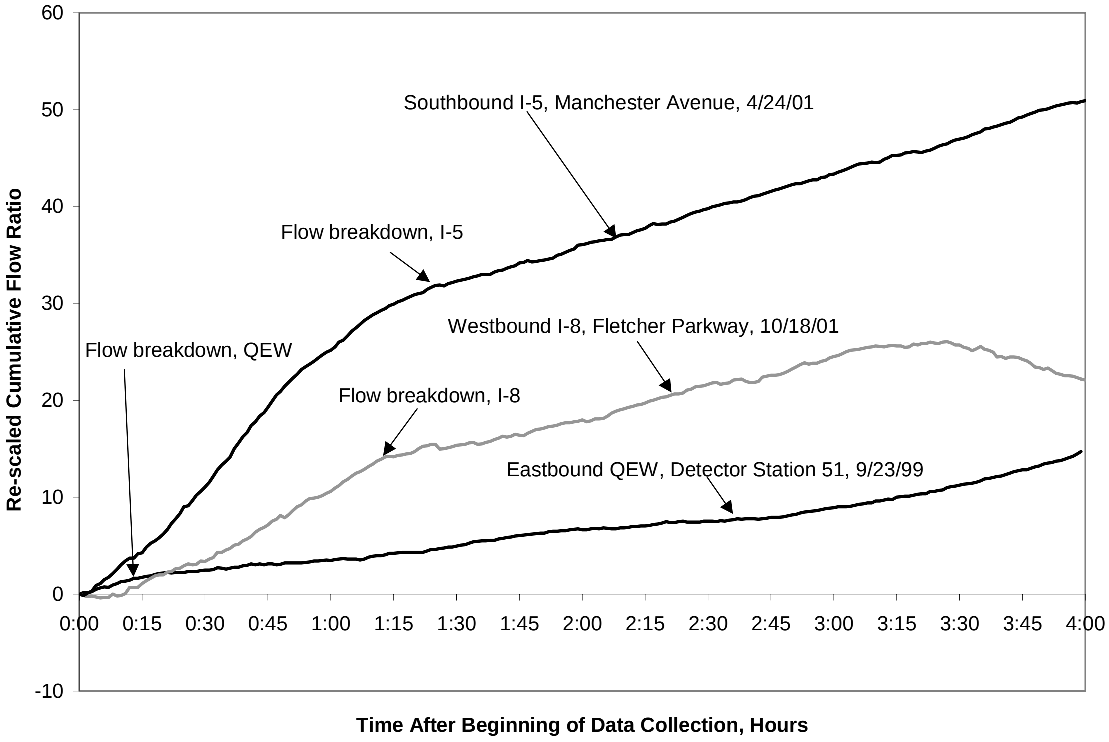


FIGURE 6 Re-scaled Cumulative Flow Ratios for Locations Upstream of the Point of Flow Breakdown



Times of flow breakdown, as determined from the speed time series, are noted on the plots.

From Figures 3 and 4, it may be seen that there were abrupt decreases in the speed ratio at about the time of flow breakdown at the San Diego sites and that there were less abrupt decreases in the flow ratios at about the same time. Note, however, that at Via de la Valle on Southbound Interstate 5, there had been a larger drop in the speed ratio about 30 minutes earlier. Following these abrupt decreases, the slopes of the re-scaled cumulative curves are approximately zero, indicating that there was no difference between the speed in the median lane and the average speed for all lanes. At the 70th Street-Lake Murray Boulevard site on westbound Interstate 8, there was another drop in the speed ratio (to an average of something less than 1.0) when a queue from downstream grew into the section later in the peak. At the site on the QEW, on the other hand, there was no change in either the speed ratio or the flow ratio at the time of flow breakdown, and only very minor changes thereafter.

Figures 5 and 6 show that at the locations farther upstream, on the other hand, there were only very minor variations in the speed ratios, and that all were greater than 1.0, with the possible exception of a period of about 30 minutes at the Fletcher Parkway site on Interstate 8. In this case, however, the speed ratio recovered long before the queue cleared. Flow ratios, on the other hand, show patterns of variation that are quite similar to those at the corresponding sites downstream.

Some of these results could be consistent with Daganzo’s theory, but others definitely are not. On the basis of the lane configurations alone, all sites in Figures 3 and 4 should be downstream of the bottlenecks. If they are also downstream of the zone of acceleration, a transition to the discharge state should be observed at the time of flow breakdown, and there should be no change in their flow and speed ratios. The unchanged speed and flow ratios at QEW Detector Station 53 are consistent with this hypothesis.

The sharp downward breaks in the slopes at the two San Diego locations could also be consistent with the theory provided the sites are not downstream of the zone of acceleration. Analysis of time series of speeds and flows, re-scaled cumulative speed curves, and changes in the number of vehicles stored in different sections (calculated by subtracting the sum of the mainline counts and the off-ramp counts at the downstream detectors from the sum of the mainline counts and the on-ramp counts at the upstream detectors) suggested that Via de la Valle was in the zone of acceleration on southbound Interstate 5 and that 70th Street-Lake Murray Boulevard was probably just upstream of the point of flow breakdown on westbound Interstate 8. Consequently, the results shown in Figures 3 and 4 could all be consistent with the theory.

On the other hand, the fact that speed ratios did not decrease at sites upstream of the points of flow breakdown, although the flow ratios did decrease, is clearly inconsistent with the theory’s underlying behavioral logic. According to the theory, flow breakdown is precipitated when speeds are equalized across the lanes. If this is the case, speed ratios should always decrease in the transition to congested flow. Also, since the rabbits are held to change lanes because the median lane no longer offers a speed advantage, there should be an abrupt decrease in the flow ratio only if speeds are equalized. Since these sites were undoubtedly congested, a major assumption of the theory is contradicted by the observed decrease in flow ratios without speed equalization.

Taken together, the results at the upstream and downstream locations seem inconsistent with one another. Why should speeds equalize at locations at or just downstream of the bottlenecks, but not farther upstream in the queue? Further analysis of the data suggests that the results at Via de la Valle and 70th Street-Lake Murray Boulevard may be the result of data biases and special circumstances, and that those in Figure 5 are not altogether representative of conditions in the queues.

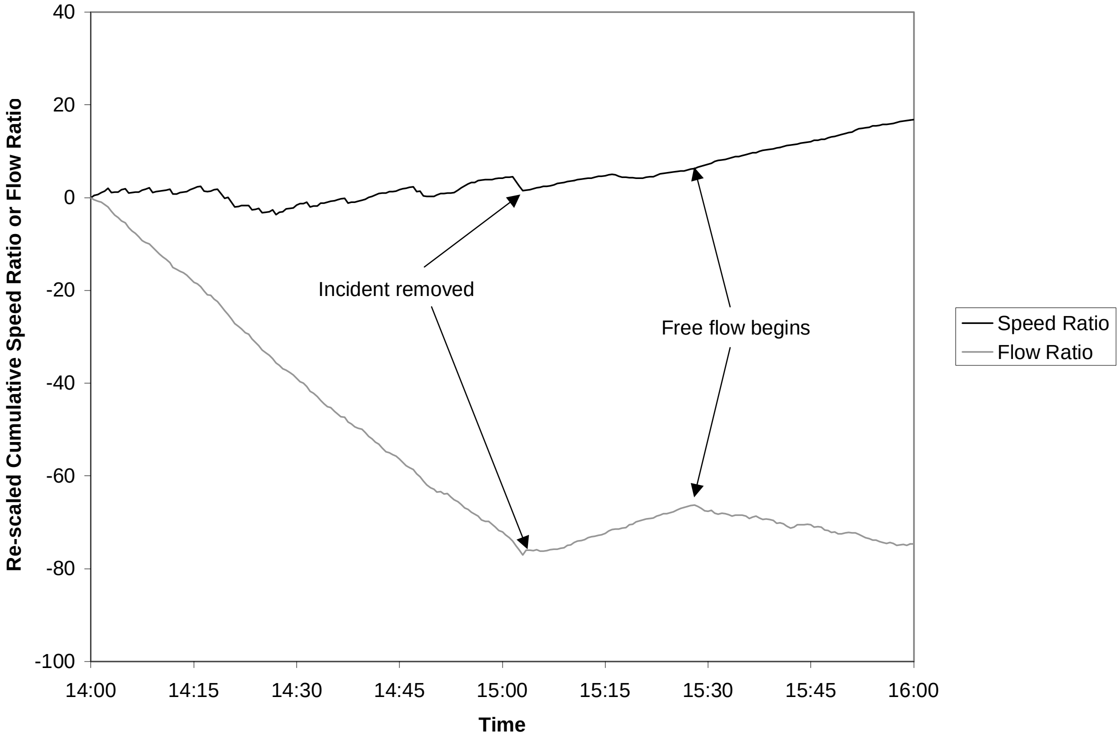
In the case of the Via de la Valle site, there is actually very little decrease in the speed ratio at the time of flow breakdown; the more pronounced decrease had taken place 30 minutes earlier. The time series of estimated speeds in the individual lanes suggests that this resulted from an increase in speed in the two outer lanes; this increase, in turn, may well have been a result of the bias in the San Diego speed estimates. Recall that these speed estimates assume a constant vehicle length and are underestimated if longer than average vehicles are present. Early in the data collection period, the outer lanes have little traffic and are used mostly by trucks. As flow increases, more passenger cars use these lanes and the bias decreases, causing the speeds in these lanes to appear to increase and the speed ratio to decrease. The Via de la Valle speed ratios may also be misleading because speeds in the median lane appear to be underestimated: they are somewhat less than those in Lane 2 even in free flow. Although the median lane speed ratio appears to be approximately 1.0 following flow breakdown, the Lane 2 speed ratio was consistently greater than 1.0.

At the 70th Street-Lake Murray Boulevard site, the time series of speeds in individual lanes shows major speed oscillations in the two left lanes following local flow breakdown, but very little oscillation in the two right lanes. During periods of minimum speed in the recurring jams, speed in the median lane is normally less than that in any of the other lanes; during periods of speed recovery, speed is highest in the median lane. This rapid oscillation in the speed ratio leads to the jagged appearance of the re-scaled cumulative speed ratio curve for this location (see Figure 3). Farther upstream, the jams spread to all lanes, and the average median lane speed ratio consistently exceeded 1.0.

Analysis of re-scaled cumulative speed ratio plots for locations upstream of Cawthra Road in the eastbound QEW queue suggests that the results in Figure 5 are not altogether representative of speed behavior in queues. Speed ratios downstream of the exit to Cawthra Road (including those at Stations 50 and 51) are consistently greater than 1.0. Farther upstream, however, those at stations upstream of exits tended to be less than 1.0 and those upstream of entrances to be greater than 1.0. This might suggest that the relative speeds in different lanes in queues are influenced by entering and exiting traffic, and that speed is rarely equalized across all lanes.

Queue discharge flow following removal of incidents was found to be reasonably similar to flow in the vicinity of merge bottlenecks. In the incident queues themselves, speed and flow ratios varied and were sometimes greater than 1.0 and sometimes less. This was to be expected, since lanes were blocked and there was roadside activity on one side of the road or the other. In queue discharge following incident removal, both speed and flow ratios were typically greater than 1.0, although there were exceptions. The exceptions included the flow ratios at Adams Avenue and El Cajon Boulevard on southbound State Route 15, and the speed ratio at College Avenue on westbound Interstate 8. In all cases, the flow or speed ratio at the location in question is typically less than 1.0, even under free flow conditions. The low speed ratio at College Avenue appears to be due to biases in the data; the low flow ratios at Adams Avenue and El Cajon Boulevard may be due to either biased data or unusual lane use in this section. Major changes in speed and flow ratios that might indicate transitions between flow states were not observed, even in cases in which there was interference from bottlenecks upstream or downstream from the incident. Figure 7 shows re-scaled cumulative flow and speed ratios for a typical incident. Note that in this case, the flow ratio was less than 1.0 in free flow following the discharge of the incident queue.

FIGURE 7 Re-scaled Cumulative Flow and Speed Ratios at 70th Street-Lake Murray Boulevard for Incident of November 29, 2001



CONCLUSION

Daganzo's theory is an important contribution to the literature of traffic flow because it attempts to relate flow phenomena to a theory of driver motivation and behavior. An underlying behavioral assumption of the theory is that aggressive drivers always act to maximize their speed; in particular, the most aggressive drivers will segregate themselves in the fastest lane so long as there are differences in speed among the lanes. The consequence of this assumption is that rapid redistribution of flow among the lanes is to be expected when speeds are equalized across the lanes (or, conversely, when they cease to be equal), but not otherwise. The theory further assumes that speeds are typically equalized among the lanes in congested flow but that speed differences will be reestablished following acceleration downstream of queues. This leads to the expectation that there will be two distinct flow states – capacity flow and discharge flow – downstream of queues.

The major findings of this study are that there are often speed differences among the lanes even in congested flow and that sudden redistribution of flow across the lanes can occur without speed equalization. These findings, in turn, imply that Daganzo's behavioral logic is oversimplified and that something more than just maximization of speed by aggressive drivers is involved in the distribution of flow across the lanes. Meanwhile, however, the study confirmed the transition to congested flow often triggers a more equal distribution of flow even when speed equalization does not occur. An obvious alternative behavioral assumption is that in choosing lanes, drivers respond to the relative density in the different lanes, as well as the relative speed.

ACKNOWLEDGEMENTS

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration. Special thanks are due to Don Day of Caltrans, Mike Mauch of the University of California Berkeley, and Chris Paolini of San Diego State university for their assistance in providing data.

REFERENCES

1. Daganzo, C. F. *A Behavioral Theory of Multi-Lane Traffic Flow Part I: Long Homogeneous Freeway Sections*. University of California at Berkeley, Institute of Transportation Studies Research Report UCB-ITS-RR-99-5, Berkeley, CA, 1999. http://www.ce.berkeley.edu/˜daganzo/BehThryPt1.pdf, accessed November 14, 2002.
2. Daganzo, C. F. *A Behavioral Theory of Multi-Lane Traffic Flow Part II: Merges and the Onset of Congestion*. University of California at Berkeley, Institute of Transportation Studies Research Report UCB-ITS-RR-99-6, Berkeley, CA, 1999. http://www.ce.berkeley.edu/˜daganzo/UCB-ITS-RR-99.6.pdf, accessed November 14, 2002.
3. California Highway Patrol. Traffic Incident Information Page. http://cad.chp.ca.gov/, accessed June 27, 2002.
4. Cassidy, M. J., and J. R. Windover. Methodology for Assessing Dynamics of Freeway Traffic Flow. In *Transportation Research Record 1484*, TRB, National Research Council, Washington, D. C., 1995, pp. 73-79.
5. Allen, B. L., F. L. Hall, and M. A. Gunter. Another Look at Identifying Speed-Flow Relationships on Freeways. In *Transportation Research Record 1005*, TRB, National Research Council, Washington, D. C., 1985, pp. 54-64.