A combined modal split and stochastic assignment model for congested networks with motorized and non-motorized transport modes

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In this paper, a network equilibrium model is proposed for the simultaneous prediction of mode choice and route choice in congested networks with motorized and non-motorized transport modes. In Hong Kong, motorized and non-motorized modes are competing travel alternatives, as the average trip length is relatively short due to high-density development in urban areas. In addition, non-motorized modes such as walking usually serve as complements to motorized trips. For example, transit passengers have to walk in order to gain access to and egress from transit stops. In this paper, the transit and walking modes are representatives of motorized and non-motorized transport modes respectively. The fundamental congestion effects of each mode and inter-modal interactions are taken into account in the simultaneous mode and route choice problem. An equivalent variational inequality (VI) problem is formulated to capture all the components of the proposed model in an integrated framework. A solution algorithm is presented for solving the VI problem. A numerical example is used to illustrate the application of the proposed model and solution algorithm. It is found that more refined estimates of travel times and network flows can be obtained using the proposed model.

INTRODUCTION

In transportation planning models, non-motorized transport modes (e.g. walking) have traditionally received less attention when compared to motorized transport modes (e.g. transit, automobiles, etc). This has led to the situation where most models are either designed to develop data for motorized modes only, or designed to represent the two modes on separate networks while neglecting inter-modal interactions. There are two possible reasons for this situation: (a) non-motorized trips represent a relatively small portion of total trips in cities in developed countries; (b) a common belief is that non-motorized trips experience no or less congestion than motorized ones.

However, due to high-density development, Hong Kong has a congested transportation system for both motorized and non-motorized modes. In addition, Hong Kong’s inherent characteristics of compact urban form and mixed land use pattern make walking a favorable transport mode. Besides, walking is necessary for nearly all motorized trips: the use of the transit implies short walking trips for access to and egress from transit stops. For the use of the car the same holds true for walking to a parking place. In order to obtain more refined estimates of travel times and network flows, (a) walking components involved in the use of motorized modes should be treated in the same manner as pure walking trips; accordingly, a motorized trip is associated with a flow-dependent walking time; (b) the fundamental congestion effect of each mode should be taken into account in the travel choice process. Hence, better solutions can be found concerning the infrastructure investment if the traditional modeling focus on motorized trips be extended to incorporate the non-motorized ones.

The network with motorized and non-motorized modes can be dealt with by using the multi-mode network equilibrium models. For the multi-mode network equilibrium problem, mode choices and route choices should be modeled simultaneously. Models that combine several travel choices are far from new. The combined (or integrated) models differ mainly in: (a) mathematical tools used to formulate the network equilibrium problem, and (b) the number and kinds of travel choices included in combined models. The combined models can be formulated by using the equivalent optimization approach (1, 2, 3, 4), variational inequality (VI) approach (5, 6) or fixed point approach (7). Despite different formulations, there is general agreement that the development of combined models should strive for balance between behavioral richness and computational tractability. The equivalent optimization approach requires strong modeling assumptions that frequently are unrealistic. However, it has computational advantages: the equilibrium problem becomes a convex optimization
problem that can be solved efficiently by existing solution algorithms. The VI and fixed-point approaches view the network equilibrium conditions as a system of equations and inequalities to be solved directly. These approaches have substantial behavioral advantages and can accommodate general demand or performance functions. But solving the general models are more difficult than solving an equivalent convex optimization problem. The linkage and equivalence conditions among these approaches were discussed by Patriksson (8).

As far as the travel choices considered in combined models, the model of Florian and Nguyen (1) combined trip distribution, modal split and assignment steps. Safwat and Magnanti (2) developed an integrated model for the simultaneous prediction of trip generation, trip distribution, modal split and assignment on large-scale networks. Lam and Huang (3) presented a multi-class combined trip distribution, mode choice and assignment model. Abrahamsson and Lundqvist (4) proposed the combined trip distribution, mode choice and assignment models with hierarchical choices, where distribution may precede mode choice or vice versa. Florian et al. (5) formulated a multi-class multi-mode variable demand equilibrium model, in which the mode choice model is a hierarchical logit function.

There is an inconsistency in the existing combined mode and route choice models. The traveler’s route choice behavior is characterized by deterministic user equilibrium (DUE), whereas his/her mode choice is governed by a logit model. Therefore, the adoption of stochastic user equilibrium (SUE) rather than DUE as the behavior principle on route choices would lead to consistency between mode choices and the route choices. This is very important for congested networks with motorized and non-motorized transport modes. In this paper, we formulate the combined model (i.e. the combined modal split and stochastic assignment model) as a VI formulation and show that its Karush-Kuhn-Tucker (KKT) conditions replicate all the model components.

This paper is organized as follows. First, some useful concepts and the multi-modal network representation are described. Second, a VI formulation of the network equilibrium problem is proposed and discussed. A solution algorithm is then proposed. A numerical example is provided to illustrate the application of the proposed model and its solution algorithm. Finally, conclusions are drawn together with recommendations for further works.

SOME USEFUL CONCEPTS AND THE MULTI-MODAL NETWORK REPRESENTATION

Some useful concepts
Motorized modes include transit, auto, etc. Of those, the transit has the most complex network profile. A transit network constitutes a set of transit lines and a set of stops (nodes) where passengers can board, alight, or transfer. A transit line can be described by the frequency and route of the vehicles as well as the vehicle types (e.g. bus or underground train). A line segment is a portion of a transit line between two consecutive stops, while a line section is any portion of a transit line between two stops that are not necessarily two consecutive nodes of its route.

Different transit lines may parallel for part of their itineraries with some stops in common. A passenger transit path (or route) is the feasible path that a passenger can follow on the transit network in order to travel between two nodes. Generally it is identified by a sequence of nodes, including the start node, the end node, and all the intermediate nodes representing the transfer nodes. The portion of a route between two consecutive nodes is associated with a set of attractive lines or common lines that is determined as described in De Cea and Fernández’s model (9). The attractive set of lines is the set of transit lines that are chosen by passengers to minimize their expected total travel time. In this paper, it is assumed that all paths chosen by passengers consist of line sections in the attractive set of lines.

Generally, a transit stop does not locate a zone centroid at which trips appear and/or disappear, so the use of the transit entails short walking trips for access to and egress from transit stops. Therefore, a transit trip is essentially a combined mode trip. Non-motorized modes consist of walking and bicycle. In Hong Kong, walking is the dominant non-motorized mode. The walking network is usually well connected; one can travel from an origin to any destination on the walking network.
The multi-modal network representation

The representation for a whole network with motorized and non-motorized modes requires less effort than generally believed. We propose a multi-layer framework in this paper. For the sake of clarity, the transit and walking modes are selected to be representatives of motorized and non-motorized modes respectively. However, the multi-layer framework can be extended to include more transport modes in a natural fashion.

Figure 1 shows the example network that consists of two modes (i.e., the transit and walking) and three zone centroids (Z1, Z2 and Z3). The transit network comprises four transit stops (A, B, C and D) and four transit lines (L1, L2, L3 and L4). This example network is the simplified network connecting Mid-level (Z1) to Central (Z3). Part of this network is served by a system of elevators that connect nodes E and F (which are termed as Mid-level elevators in this paper). The location map is shown in Figure 2 together with photos of the Mid-level elevators (linking node E and F) and Central Market (Z2).

The multi-layer network is constructed in such a way that each single-modal network is represented on a separate layer and various layers are connected by transfer links, as shown in Figure 3. On the transit layer, the transit network is represented with respect to transit links (or route sections, denoted by S1-S6). This representation for a transit network was defined by De Ceà and Fernández (9), for simplifying the calculation of common lines. Note that a transit trip is completed by using walking links, transfer links, and transit links. In contrast, a pure walking trip will only uses walking links.

The merits of the multi-layer network can be summarized as follows: (a) the structure of the multi-layer network is flexible; (b) the multi-layer network behaves like a simple network; (c) combined mode trips and inter-modal interactions can be modeled based on the multi-layer network.

MODELS FORMULATION

Definitions

In order to present the model components and formulation, the following definitions are used. Let \( G = (N, S) \) be a directed network defined by a set \( N \) of nodes and a set \( S \) of directed links. Let \( M \) be the set of modes. The single-modal network of the generic mode \( m \in M \) is represented by a subset of \( G \) denoted \( G^m = (N^m, S^m) \), \( N^m \subseteq N \), \( S^m \subseteq S \). The demand for travel by mode \( m \) between origin-destination (O-D) pair \( w \), \( w \in W \), is denoted by \( g_w^m \), where \( W \) is the set of O-D pairs. Denote by \( g_w \) the total demand for travel between O-D pair \( w \). Denote by \( h_r^w \) the traveler flow on route \( r \), \( r \in R_w \) between O-D pair \( w \), where \( R_w \) is the set of routes between O-D pair \( w \). Let \( R^m_w \) be the set of routes by mode \( m \) between O-D pair \( w \), i.e., \( R_w = \bigcup_{m \in M} R^m_w \). Let \( v_s \) be the traveler flow on link \( s \). Each link \( s \in S \) has an associated flow-dependent cost \( c_s \), which denotes the cost per unit flow or average cost on that link. Let \( c_r^w \) denote the travel cost on route \( r \) between O-D pair \( w \), we have \( c_r^w = \sum_{s \in r} c_s \cdot \alpha_s \), where \( \alpha_s \) is the element of the link-route incidence matrix, which equals 1 if link \( s \) lies on route \( r \), 0 otherwise.

The VI formulation of the combined modal split and stochastic assignment problem

The combined modal split and stochastic assignment problem is the simultaneous prediction of modal split between origins and destinations in a multi-modal network and stochastic assignment of trips to routes in each O-D pair. The feasible set \( \Omega \) of the demands for modes and route flows is defined by:

\[
(I^m_w) \quad \sum_{m \in M} h_r^w = g_w^m \quad m \in M \quad w \in W \\
(I_w) \quad \sum_{m \in M} g_w^m = g_w \quad w \in W \\
(\lambda^w) \quad h_r^w \geq 0 \quad r \in R_w \quad w \in W \\
\quad g_w^m > 0 \quad m \in M \quad w \in W
\]
where the dual variables, which will be used later in the analysis, are indicated in parentheses. Equations (1) and (2) are route flow and mode demand conservation constraints. Constraints (3) and (4) are the usual nonnegative and positive requirements on route flows and mode demands respectively. The total demand is fixed in this paper.

The VI formulation of the combined modal split and stochastic assignment problem is to find \((\mathbf{h}^*, \mathbf{g}^w) \in \Omega\) which satisfies

\[
\sum_{s \in S} \sum_{w \in W} \left( \sum_{r \in R_s^w} \left( c_{w}^s (\mathbf{h}^w \cdot \mathbf{g}^w) (h_{w}^s - h_{w}^s_m) + \frac{1}{\theta} \ln (h_{w}^s / g_{w}^m) \cdot (h_{w}^s - h_{w}^s_m) \right) + \frac{1}{\theta} \ln (g_{w}^m) \cdot (g_{w}^m - g_{w}^m) \right) \geq 0
\]

(5)

where it is defined that \(0 \ln 0 = 0\), \(\theta_r\) is the positive calibrated parameter which is used to measure the cost sensitivity on route choices: as \(\theta_r \rightarrow 0\) route choices become equiprobable, and as \(\theta_r \rightarrow +\infty\) route choices become extremely concentrated on the least-cost route in each set \(R_s^w\); \(\theta_m\) is the positive calibrated parameter which is used to measure the cost sensitivity on mode choices: as \(\theta_m \rightarrow 0\) mode choices become equiprobable, and as \(\theta_m \rightarrow +\infty\) mode choices become extremely concentrated on the least-cost mode of available modes for each O-D pair; the feasible set \(\Omega\) is defined by constraints (1)-(4), and \((\mathbf{h}, \mathbf{g}^w)\) are the vectors of route flows and mode demands respectively. For the model to be internally consistent, \(\theta_r \geq \theta_m > 0\) must hold (10). This formulation looks similar to the model proposed by Florian et al. (5), where DUE rather than SUE is adopted as the travelers’ behavior on route choices.

Now we derive and analyze the KKT conditions of the VI formulation (5), which are given below:

\[
\begin{align*}
\left( h_{w}^s \right) & : \quad c_{w}^s + \frac{1}{\theta} \ln h_{w}^s / g_{w}^m - l_{w}^m - \lambda_{w} = 0 \quad r \in R_s^w \quad m \in M \quad w \in W \\
\left( g_{w}^m \right) & : \quad \frac{1}{\theta} \ln g_{w}^m + l_{w}^m - l_{w} = 0 \quad m \in M \quad w \in W
\end{align*}
\]

(6)

(7)

Complementarity:

\[
\begin{align*}
h_{w}^s \lambda_{w} & = 0 \quad r \in R_s^w \quad w \in W \\
\lambda_{w} & \geq 0 \quad r \in R_s^w \quad w \in W
\end{align*}
\]

(8)

(9)

and (1)-(4).

The form of (6) ensures that \(h_{w}^s > 0\), so we have

\[
h_{w}^s / g_{w}^m = \exp (\theta) (\sum_{s \in S} \exp (- \theta c_{w}^s)) = 1 \quad m \in M \quad w \in W
\]

(10)

Combining equations (1) and (10) leads to

\[
\exp (\theta) l_{w}^m \sum_{s \in S} \exp (- \theta c_{w}^s) = 1 \quad m \in M \quad w \in W
\]

(11)

Hence, the following logit route choice model can be deduced by combining equations (10) and (11)

\[
h_{w}^s = g_{w}^m \cdot \exp (- \theta c_{w}^s) / \sum_{s \in S} \exp (- \theta c_{w}^s) \quad r \in R_s^w \quad m \in M \quad w \in W
\]

(12)

The logit modal split model can be deduced by combining equations (2) and (7)

\[
g_{w}^m = g_{w}^m \cdot \exp (- \theta c_{w}^s) / \exp \sum_{s \in S} (- \theta c_{w}^s) \quad m \in M \quad w \in W
\]

(13)

where \(l_{w}^m\) can be obtained from equation (11):

\[
l_{w}^m = - \frac{1}{\theta} \ln \left( \sum_{s \in S} \exp (- \theta c_{w}^s) \right) \quad m \in M \quad w \in W
\]

(14)

Therefore, the proposed VI formulation (5) does indeed lead to a combined modal split and stochastic assignment model, or, in other words, the application of the hierarchical logit model to represent the joint probabilities of simultaneous mode and route choices.

A standard theorem in the theory of variational inequalities indicates that: if all the functions entering the VI formula are continuous and the feasible set \(\Omega\) is compact, then there exists at least one solution to VI formulation (5). The feasible set \(\Omega\) is defined by linear constraints, positive and nonnegative constraints, so \(\Omega\) is
non-empty, closed and convex. Due to the fixed (or bounded) O-D demand, the feasible set $\Omega$ is compact. Obviously, $(1/\theta_m)\ln h_{\theta_m}$ and $(1/\theta_m)\ln(g_{\theta_m})$ are continuous. Therefore, VI formulation (5) has at least one solution if $c_\theta$ (or equivalently, $c_\theta$) is continuous. This condition is very mild and the link cost function $c_\theta$ is continuous in many realistic settings. When a certain monotonic condition is satisfied, Dafermos (11) showed that VI problems have unique solutions. However, the monotonic condition, in general, may not be satisfied in the multi-mode network equilibrium problem and there is no guarantee of a unique solution in this case.

**Capacity restraint in transit services and bi-directional flow effects on walkways**

On a congested transit network, a proportion of passengers may not be able to get on the first arriving vehicle at some busy stops. These passengers have to be served by the next coming vehicle or transfer to alternative lines. This phenomenon is due to the fact that each transit link has an absolute capacity that cannot be exceeded in practice. Lam et al. (12) proposed a transit SUE assignment model to consider capacity restraint explicitly, by imposing the constraint:

$$v_s \leq k_s, \ s \in S^b$$

where $k_s$ is the capacity of transit link $s$, $S^b$ is the set of all transit links. When the constraint (15) is reached, it was proved by Lam et al. (12) that the associated Lagrange multipliers are equivalent to the equilibrium passenger overload delays on the congested transit network. Therefore, the travel time on transit links consist of three terms: the in-vehicle travel time (denoted as $\tilde{t}$), the waiting time (denoted as $u$) and passenger overload delay (denoted as $d$) if the capacity restraint of transit links is reached. Transit fare is neglected in this paper. Note that the constraint (15) is a linear inequality. The feasible set $\Omega$ is also compact if this constraint is introduced.

The greatest challenge in modeling a walking network is that, unlike roadways where the vehicle flow is separated by direction, bi-directional walkways are the norm rather than the exception. When pedestrians are walking on a bi-directional walkway facing heavy opposing walking flows, they usually weave through the opposing pedestrians with little freedom to choose walking manner (stride length, frequency of maneuver, etc). Thus, the capacity of the walkway and pedestrian walking speeds would be reduced, especially in the minor flow direction (13,14).

Based on empirical results, Chan et al. (15, unpublished data) proposed a generalized walking time function to take account of bi-directional flow effects on walkways at various flow conditions, ranging from free flow to congested situations. The generalized walking time function is:

$$t_{w^s} (v_s, v_w) = t_0 + B[v_s + v_w] + [v_s + v_w] - [v_s + v_w]^{1/\alpha}, \ a \in A$$

where a physical walkway $a$ is represented by two walking links denoted as $a^+$ and $a^-$, $t_{w^s}$ is unit walking time (second/meter) in direction (+) on physical walkway $a$ with bi-directional flows, $t_0$ is unit walking time at free flow condition (second/meter) on physical walkway $a$, $K_a$ is the capacity of the physical walkway $a$ under uni-directional flow conditions (pedestrian/meter/second), $B, s, y$ are parameters to be calibrated with observed data. $A$ is the set of physical walkways.

Clearly, the Jacobian matrix of generalized walking time functions is asymmetric. This makes it necessary to use a VI formulation for the network equilibrium problem. In other words, the network equilibrium problem is formulated as VI formulation (5) so to accommodate the asymmetric generalized walking time function. The positive definiteness of the Jacobian is dependent on the parameters of the walking time function. The inclusion of the asymmetric generalized walking time function (16) into network equilibrium models leads to models without the guarantee of a unique solution. Through transformation, the generalized walking time function (16) can be expressed as $t_{w} (v_s, v_w) = t_0 + B[v_s + v_w] + [v_s + v_w]^{1/\alpha}$. We define the effective capacity of a walking link as $\hat{K}_s(v) = K_s - B[v_s + v_w] - [v_s + v_w]^{1/\alpha}$, so the generalized walking time function (16) is expressed as $t_s = t_0 + B[v_s + \hat{K}_s(v)]$, $s \in S^p$, where $S^p$ is the set of walking links, i.e. $|S^p| = 2|A|$. $v$ is the walking link flow pattern. Walking time on walking links and the equilibrium walking link flow pattern are related to the effective capacity $\hat{K}_s(v)$. On the other hand, the effective capacity $\hat{K}_s(v)$ is
dependent on the walking link flow pattern. Due to this relationship between the effective capacity and the walking link flow pattern, an iterative procedure is employed to solve the walking SUE assignment with asymmetric generalized walking time functions.

**SOLUTION ALGORITHM**

A solution algorithm is proposed for solving VI formulation (5). Algorithms to solve (generalized) VI problems can be found in Patriksson (16), where the cost approximation (CA) algorithm is presented for solving these problems. The proposed solution algorithm can be visualized as an instance of this CA algorithm. The convergence of the proposed solution algorithm is heavily dependent on the parameters of the adopted walking time function.

The Block Gauss-Seidel decomposition method (equilibration algorithm), is used in this paper where the network assignment for given demands is computed in one block and the modal split model is solved in the other block. The demand of each mode is computed at each successive iteration by using the method of successive averages (MSA) over route travel costs. For the network assignment block, modes are solved in sequence by the use of the nonlinear Jacobi method. Transit trips and walking trips are distributed among transit routes and walking routes, respectively. A transit trip includes some walking components and thus is associated with a flow-dependent walking time.

In order to account for the bi-directional effects of walkways, two iterative processes are employed in the SUE assignment for walking trips: the inner iteration for the walking SUE solution with the assumed effective capacity and the outer iteration for the solution of the effective capacity. Lam et al.’s (12) iterative balancing procedure is used to solve the transit SUE assignment with capacity restraint. The KKT conditions of VI formulation (5) are used both in the network assignment block and in the modal split block.

The iterative balancing procedure (12), which is used to deal with the explicit capacity restraint on transit links, requires explicit route information. In addition, route travel costs are necessary for computing the log-sum and modal split model (13)~(14). Therefore, the proposed solution algorithm works with sets of predefined or “known a priori” routes. This has the advantage of explicitly identifying those routes that would likely be used. Further, predefining the working routes reduces the load substantially (17). In situations where travelers use any routes in the network without restrictions or preferences, the proposed solution algorithm can be adapted and coupled with a column generation procedure (18, 19, 20). The procedure of the solution algorithm is shown in Figure 4, where the 1st loop are unnecessary in the case that routes could be derived on behavior grounds and hence constitute sets of likely used routes.

**NUMERICAL EXAMPLE**

An example is provided to demonstrate the application of the proposed model and its solution algorithm. Specifically, the general convergence behavior of the proposed algorithm and the performance of the model are investigated together with the assessment of impacts for various model parameters (especially, $\theta_r$ and $\theta_m$).

The example network is the same with that in Figure 1, and then is represented by the multi-layer network shown in Figure 3. Transit line data are given in Table 1. The adopted walking time function is:

$$\tau_{vw}(v_i, v_f) = 0.5368 + 0.7694 \cdot \{\sqrt{1.4 \cdot \{v_{e/c} / \sqrt{v_{e/c} + v_{f/w}}\}} 0.2412 / 5.7069\}^{0.7069} \tag{17}$$

Parameters of walking links are listed in Table 2. The Mid-level elevators connect nodes E and F where a travel time of 25 minutes from E to F and 30 minutes from F to E is assumed. The steady state assumption is adopted. The basic O-D demands (traveler/h) used in the example are as following:

$$Z_1 \rightarrow Z_3 = 400, Z_2 \rightarrow Z_3 = 200, Z_3 \rightarrow Z_1 = 300, Z_3 \rightarrow Z_2 = 100.$$ 

For the sake of facilitating the essential ideas in this paper, we work with “known a priori” routes in this example.

The proposed algorithm exhibits a good convergence behavior in all tests. Figure 5 shows the convergence of the modal split for $\theta_r=0.5$ and $\theta_m=0.1$. The modal split is indicated by the ratio of the number of transit trips to that of total trips. We see that the modal split stabilizes after several iterations.
Results for the example network with various $\theta_r$ are reported in Table 3. The value of $\theta_m$ is fixed to be 0.1 in this test. The alternative routes connecting $Z_1$ to $Z_2$ are listed in Table 3, where transit links ($S_1 \sim S_6$) are bold. A transit trip is characterized by following on a route consisting of walking links, transfer links, and transit links. Therefore, the travel time of a transit trip contains a walking time component that is denoted in parentheses.

We can see from Table 3 that route flows vary with the value of $\theta_r$ and travelers are increasingly concentrated on routes with lower travel time as $\theta_r$ increases. $\theta_r \rightarrow \infty$ will approximate DUE solutions where travelers will be concentrated on routes with the least travel time. Therefore, the adoption of SUE as travelers’ route choice behavior makes it possible to calibrate the model and to simulate the observed route flow pattern with less error. Of transit routes, route R4, which contains transit link $S_1$, is the fastest route connecting $Z_1$ to $Z_2$. More travelers would choose route R4 as $\theta_r$ increases. The flow on it reaches the capacity of transit link $S_1$ if $\theta_r = 0.5$. In this case, the passenger overload delay appears for boarding transit link $S_1$, which keeps the flow on transit link $S_1$ from exceeding its capacity. In addition, the walking time involved in the use of the transit is dependent on the traffic conditions on the walking network. Thus, the decrease of the walking time for access to and egress from transit stops will reduce the total travel time of a transit trip, attracting more travelers to use the transit.

Figure 6 shows the resultant modal split for different parameters, where parameter $\pi$ indexes the demand level (or congestion level): a specific $\pi$ corresponds to the O-D demands that is the basic O-D demands multiplied by the value of $\pi$. It can be seen that the modal split is more dependent on the value of $\theta_m$ than that of $\theta_r$, even though the value of $\theta_r$ can affect the log-sum of each mode (14) and thus the modal split. Setting $\theta_m = \theta_r$ will result in a model similar to that presented by Sheffi and Daganzo (21), where mode choices are modeled as part of the route choice model. In this case, the resultant modal split may be inconsistent with the actual situation. As such, it can be seen that the proposed model has added functionality since it has two dispersion parameters (i.e., $\theta_m$ & $\theta_r$) to measure the cost sensitivity on mode choices and route choices separately. From Figure 6, we can see that the modal split also varies with different demand levels ($\pi$). This indicates that the proposed model can capture the congestion effect of each mode and inter-modal interactions.

CONCLUSIONS

In this paper, a network equilibrium model is proposed for the simultaneous prediction of mode choice and route choice in congested networks with motorized and non-motorized transport modes. The motorized and non-motorized modes is modeled jointly and on the same level, yielding a model that can predict travel choices while accounting for the fundamental congestion effect of each mode and inter-modal interactions.

An equivalent variational inequality (VI) problem is formulated to capture all the components of the proposed model in an integrated framework. The VI formulation is proved to have at least one solution and can be obtained by the proposed solution algorithm. The adoption of SUE as travelers’ route choice behavior not only leads to consistency between mode choices and route choices, but also makes it possible to calibrate the model.

The capacity restraint on transit links and bi-directional effect on walkways are taken into account in the proposed formulation. As a result of inter-modal interactions, the walking time involved in the use of the transit is dependent on the traffic conditions on the walking network. Thus, the decrease of the walking time for obtaining access to and egress from transit stops will reduce the total travel time of a transit trip, attracting more travelers to use the transit.

However, it should be noted that since the asymmetric generalized walking time function is involved; the convergence to a unique solution cannot be guaranteed. Although the proposed algorithm exhibits a good convergence behavior in tests, further study is required to assess for the convergence of the algorithm when applied to real-world large-scale transportation networks.

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<td>Effective width (m)</td>
<td></td>
<td>9</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Length (m)</td>
<td></td>
<td>400</td>
<td>400</td>
<td>1500</td>
<td>1500</td>
<td>400</td>
</tr>
</tbody>
</table>
TABLE 3  Results for the Example Network with Various $\theta_r$

<table>
<thead>
<tr>
<th>O-D pair</th>
<th>Route</th>
<th>$\theta_r=0.1$</th>
<th>$\theta_r=0.5$</th>
<th>$\theta_r=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Flow</td>
<td>Travel time</td>
<td>Flow</td>
</tr>
<tr>
<td>$Z_1 \rightarrow Z_3$</td>
<td>$R_1={Z,E, A, B, C, D, G, Z_3}$</td>
<td>35</td>
<td>61.4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$R_2={Z,E, F, G, Z_3}$</td>
<td>125</td>
<td>48.1</td>
<td>186</td>
</tr>
<tr>
<td></td>
<td>$R_3={Z,E, A, B, C, F, G, Z_3}$</td>
<td>23</td>
<td>66.2</td>
<td>0</td>
</tr>
<tr>
<td>$Z_4 \rightarrow Z_3$</td>
<td>$R_4={Z,E, A, A', S_1, D', D, G, Z_3}$</td>
<td>67</td>
<td>55.1(21.1)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$R_5={Z,E, A, A', S_2, S_5, D', D, G, Z_3}$</td>
<td>60</td>
<td>56.1(21.1)</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>$R_6={Z,E, A, A', S_3, S_4, S_5, D', D, G, Z_3}$</td>
<td>45</td>
<td>59.1(21.1)</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>$R_7={Z,E, A, A', S_3, S_4, S_5, D', D, G, Z_3}$</td>
<td>45</td>
<td>59.1(21.1)</td>
<td>103</td>
</tr>
</tbody>
</table>
Z1: A centroid
Z2: A node
L2(1): A walking link
L2(2): The $i^{th}$ segment of transit line $L_2$

FIGURE 1 The example multi-modal network.
FIGURE 2  The location map and photographs.
FIGURE 3 The multi-layer network.
<table>
<thead>
<tr>
<th>Loop</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Input: Network data and demand data</td>
</tr>
<tr>
<td>2nd</td>
<td>Generate routes of each mode at the current solution to the network equilibrium problem</td>
</tr>
<tr>
<td>3rd</td>
<td>Perform the modal split according to the logit mode choice model</td>
</tr>
<tr>
<td>4th</td>
<td>Assign the transit demand to the corresponding transit routes according to the logit route choice model with taking capacity constraint into account (using iterative balancing procedure)</td>
</tr>
<tr>
<td>5th</td>
<td>Obtain the effective capacity of each walking link</td>
</tr>
<tr>
<td></td>
<td>Assign the walking demand to the corresponding walking routes according to the logit route choice model</td>
</tr>
</tbody>
</table>

Output: route flows and travel time, the demand of each mode, walking time involved in transit trips

**FIGURE 4** The procedure of the solution algorithm.
FIGURE 5  The convergence of the modal split.
FIGURE 6  The resultant modal split for different parameters.