Routing and Scheduling for Hazardous Material Shipments 
on Networks with Time Dependent Travel Times

Ali Haghani
Professor
Department of Civil and Environmental Engineering
University of Maryland
College Park, Maryland, 20742-3021
(301) 405-1963 (Phone)
(301) 405-2585 (Fax)
E-Mail: haghani@eng.umd.edu

Yin-Jung Chen
Department of Civil and Environmental Engineering
University of Maryland
E-mail: yjchen@wam.umd.edu

Transportation Research Board
January 2003

Word Count
Text = 5241
Figures and Tables = 2250
Total = 7491
ABSTRACT

With the growth of industrialization, the demand for hazardous material shipment is quickly increasing. The safety of hazardous material shipments is an important issue to the public. Previous researchers focused attention on minimizing the risk of the route and the transportation cost. In fact, there is an important part missing in this puzzle. Since the hazardous material shipments are usually interstate and very long, most of the shipments require more than one day to be completed. The rest nodes where the drivers rest and refuel are also potential danger points to the public. Also, in real world, travel times over the links vary by time of day.

An alternative way to solve hazardous material transportation problem is presented which simultaneously considers three factors: risk of the route, risk of the rest nodes and travel times. Most current approaches assume static conditions that might result in sub optimal routes. This study proposes an integrated routing and scheduling approach to solve this problem. The paper presents a mathematical formulation, a solution approach, and test results.

Key Words: Hazardous Materials, Routing, Scheduling, Optimization, Heuristic, Location Network
1. INTRODUCTION

Transporting hazardous materials to either the destination deposit sites or other plants for further processing is an inevitable and serious problem. Many different types of cargo are classified as hazardous including but not limited to gasoline, chemical material or waste, radioactive material, explosive liquids, etc. Since these materials are mostly dangerous and harmful to both human beings and the natural environment, a carefully selected route that involves less risk is necessary to prevent accidents, or, if they happen, to minimize their impact.

The route for hazardous material transportation must represent a compromise between the internal cost (for the company or the organization that wants to ship the hazardous material) and the social cost (the potential risk cost to the society). As a matter of fact, these two factors usually conflict with each other. Therefore, multiple criteria are necessary for analyzing the route options. The goal of the analysis is to find an optimal route that balances all of the important criteria. It should be noted that there is no single best route, because the weight of each criterion can vary to meet different requirements.

Some hazardous material shipments are interstate shipments that are long distance, and therefore, these shipments are more vulnerable and dangerous. Because of the fuel and drivers’ workload constraints, these shipments require more than one day to be delivered. Therefore, one or more refueling or rest sites, depending on the length of the path, must be selected along the route. For the sake of reducing public risk, these intermediate stop sites must be selected carefully at the same time as the routing for these hazardous shipments.

Besides, during weekdays, the traffic conditions on a transportation network follow a similar pattern every day. Since this pattern can be predicted based on historical data, we can define a time-varying transportation network instead of a static network. The importance of time-varying transportation networks arises because the travel time depends not only on the linear distance of the road but also on when it is visited. We can take advantage of traveling during non-peak hours on major roads to reduce travel time and risk. By scheduling the departure time carefully, both risk and cost of each shipment can be reduced, and the safety of each shipment can be ensured.

The objective of this study is minimizing the public risk and private cost simultaneously by routing the trucks and scheduling the departure times, while several intermediate stops are made on the path from origins to destinations so that drivers can rest and trucks can be refueled. The departure time scheduling process will take place in the origin and all intermediate stops. The key issue here is considering an integrated routing, scheduling, and location approach so that the drivers can spend less travel time on the road and avoid the high risk links and nodes. The safety of each shipment can be improved by this integrated approach.

2. LITERATURE REVIEW

Since late 1970s, the hazardous material transportation has attracted researchers’ attentions in the areas of risk analysis, routing / scheduling and facility location. Several improvements have been proposed over the past decades, such as from single objective function to multiple objective function, from pure route risk to population exposure risk, from simple routing to integrated routing and location/scheduling, and from fixed risk to time-dependent risk.
The easiest algorithm for routing is the classical shortest path routing, since it minimizes a single measure on the transportation network to find the cheapest route from the origin to the destination. This approach was used to solve hazmat transportation problem in earlier stages (Joy et al., 1981, and Brogan and Cashwell, 1985). Risk assessment is an important issue. The most commonly used measure of risk is population exposure, which is an easier way to compare the risk values between different routes. Population exposure could be expressed in many different ways, and different studies used different equations such as Glickman (1983) and Pijawka et al. (1985), but the main idea is still the same.

Shobrys (1981) considered two objectives, which are (1) Minimize ton-miles traveled. (2) Minimize population exposure-ton. Suljoadkusumo and Nozick (1998) presented work. They used these three criteria to find non-dominated routes and compared the risk-cost tradeoff among the routes.

Recently, Miller-Hooks and Mahmassani (1998) presented a stochastic, time varying transportation model. The paper is concerned with the selection of routes in a network along which to transport hazardous materials, taking into consideration both the length of time in transit and the risk of population exposure in the event of an incident. The risk is estimated by the population exposure on the path and the cost is estimated by the travel time.

The shortest route through a network with time-dependent travel time was presented by Cooke and Halsey in 1966. Since the real world transit time is time dependent, this new problem can provide a better solution which is more practical and reflects the real world condition. Nozick et al. (1997) presented the idea of routing and scheduling hazmat transportation in a time dependent transportation network. Since the condition of the route changes over time, this paper proposed a method to capture this special feature and to take advantage of it.

The Multi-Criterion Shortest Problem (MSP) is a shortest path problem with more than one criterion; travel time or travel distance is most commonly used in conventional single objective shortest path problem. The MSP deals with a network in which each arc possesses two or more characteristics simultaneously. Orda and Rom (1991) presented a minimum weight path model in a time dependent network.

3. MODEL FORMULATION

Hazardous material transportation problem can be deemed as a special case of shortest path problem. Considering a given origin and a destination in a time dependent transportation network, this problem can be formulated as a multi-criterion shortest path problem (MSPP) with several intermediate stops.

Due to the nature of the problem, an algorithm for Time-Dependent Shortest Path with Intermediate Stops (TDSPI) problem is required. This section presents a mathematical formulation for the TDSPI. This formulation will help us determine the optimum route and departure schedule from the origin and each intermediate node.

Let \( G[N, A, C(t)] \) denote a time dependent network, where \( N = \{ 1, 2, \ldots, n \} \) is a finite set of nodes, \( A = \{ a_1, a_2, \ldots, a_m \} \) is a finite set of arcs, and \( C(t) \) is the time dependent cost (travel time) of each arc. Furthermore, let the 1st node, Node 1, be the origin node and the nth node, Node n, be the destination node.

We want to find the optimal route that has the shortest travel time and the least risk, and the corresponding schedule for the route. In other words, we want to find a schedule of arrival times and departures time at the nodes so that the overall objective value is minimum. This
operation is a sequence of arrivals and departures. The flowchart in Figure 1 shows the sequence of this operation.

The following assumptions are made in the proposed formulation.

- The travel time of each link has an identical and independent pattern and repeats itself cyclically.
- Link capacity is ignored.
- The driver has his daily maximum workload.
- The population at risk will increase as the driver’s rest time increases at the rest node.
- The truck is refueled at the same node where the driver rests.

The following notation is used in the formulation.

\[ P_1: \text{Weight of objective 1.} \]
\[ P_2: \text{Weight of objective 2.} \]
\[ P_3: \text{Weight of objective 3.} \]
\[ F_1: \text{Objective 1.} \]
\[ F_2: \text{Objective 2.} \]
\[ F_3: \text{Objective 3.} \]
\[ \text{Risk}_{link}^{ij}: \text{Link accident probability.} \]
\[ \text{Risk}_{node}^{i}: \text{Node accident probability.} \]
\[ \text{pop}_{link}^{ij}: \text{Resident population density within impact width of link } (i, j). \]
\[ \text{pop}_{node}^{i}: \text{Resident population density within impact area of node } i. \]
\[ i: \text{Index for nodes } i \in \mathcal{N}. \]
\[ d: \text{Index for days; } d = 1, 2, 3, \ldots. \]
\[ q: \text{a large number.} \]
\[ k: \text{Driver’s workload per day.} \]
\[ x_{ij}: \begin{cases} 1, & \text{if the link } (i, j) \text{ is in the path.} \\ 0, & \text{otherwise} \end{cases} \]
\[ a_i: \text{Arrival time at node } i. \]
\[ t_{ij}: \text{Cumulative travel time at link } (i, j). \]
\[ w_i: \text{Cumulative work hours from prior nodes to node } i. \]
\[ h_{ij}: \text{Cumulative work hour at link } (i, j). \]
\[ y_i^d: \begin{cases} 1, & \text{if node } i \text{ is visited after the } d^{th} \text{ day.} \\ 0, & \text{otherwise.} \end{cases} \]
\[ z_i^d: \begin{cases} 1, & \text{if node } i \text{ is visited on the } d^{th} \text{ day.} \\ 0, & \text{otherwise.} \end{cases} \]
\[ \text{Day}_i^d: \text{Arrival time at node } i \text{ on the } d^{th} \text{ day.} \]
\[ \text{Day}_i^d = \begin{cases} a_i, & \text{if node } i \text{ belongs to the } d^{th} \text{ day.} \\ 0, & \text{otherwise.} \end{cases} \]
\[ L_i^d: \text{The latest arrival time at a node on the } d^{th} \text{ day.} \]
\[ s_i^d: \text{Indicator of } L_i^d. \]
\[ s_i^d = \begin{cases} 0, & \text{if } i \text{ is the rest node in a day} \\ 1, & \text{otherwise.} \end{cases} \]
RestDay\(^d\): Indicator of rest day.

\[
\text{RestDay}^d = \begin{cases} 
1, & \text{if drivers must rest on the } d\text{th day.} \\
0, & \text{otherwise.}
\end{cases}
\]

\(R_i^d\): rest hours at node \(i\) on the \(d\)th day.

\(u_{ij}^t\): 1, if vehicle enters link \((i, j)\) at time \(t\).

\(0\), otherwise.

\(v_{ij}^t\): travel time of link \((i, j)\) at time \(t\).

\(m_i^d\): 1, if node \(i\) is a rest node on the \(d\)th day.

\(0\), otherwise.

\(c_{ij}\): Travel time of link \((i, j)\)

Before we introduce the proposed model, we have to discuss two important issues. The first one is the representation of the cumulative travel time of link \((i, j)\) and how it can be incorporated into the constraints of the model. The cumulative travel time on link \((i, j)\) can be expressed as follows:

\[
t_{ij} = \begin{cases} 
a_i + \sum_d R_i^d + c_{ij} (a_i + \sum_d R_i^d) & \text{if } x_{ij} = 1 \\
0 & \text{otherwise}
\end{cases}
\tag{3.1}
\]

where \(C_{ij}(t)\) is a function that determines travel time of link \(ij\) at time \(t\).

Obviously, (3.1) is not a valid equation in a strict mathematical optimization sense. Therefore, we rewrite this relationship in three parts, (3.2), (3.3) and (3.4). These new equations can easily be embedded in a mathematical formulation and are CPLEX compatible. CPLEX is an of-the-shelf software for solving mixed integer linear programming problems. Constraint (3.2), (3.3), and (3.4) ensure that the \(t_{ij}\) will be equal to the arrival time at the start node plus the rest hours at the start node plus the travel time of link \((i, j)\) only when link \((i, j)\) is in the path.

\[
q \cdot (1 - x_{ij}) + t_{ij} - a_i - c_{ij} - \sum_d R_i^d \geq 0, \quad \forall (i, j) \in \mathcal{A}
\tag{3.2}
\]

\[
q \cdot x_{ij} - t_{ij} \geq 0
\tag{3.3}
\]

\[
a_i + \sum_d R_i^d + c_{ij} - t_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}
\tag{3.4}
\]

The second issue is the calculation of the cumulative workload at node \(i\). The variable \(h_{ij}\) represents the cumulative work hours through link \((i, j)\). This variable is a positive integer only when link \((i, j)\) is selected in the route as shown below.

\[
h_{ij} = \begin{cases} 
w_i + c_{ij} (a_i + \sum_d R_i^d) & \text{if } x_{ij} = 1 \\
0 & \text{otherwise}
\end{cases}
\tag{3.5}
\]

It can be rewritten using the following three inequalities.

\[
q \cdot (1 - x_{ij}) + h_{ij} - w_i - c_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{A}
\tag{3.6}
\]
\[ q \cdot x_{ij} - h_{ij} \geq 0 \]  \hspace{1cm} (3.7)

\[ w_i + c_{ij} - h_{ij} \geq 0 \hspace{0.5cm}, \forall (i, j) \in A \]  \hspace{1cm} (3.8)

The mathematical formulation of the hazmat transportation problem is as follows.

\begin{align*}
\text{Min } F & = P_1F_1 + P_2F_2 + P_3F_3 \\ 
\text{Subject to} & \\
F_1 & = a_n \\
F_2 & = \sum_{i} \sum_{j} x_{ij} \cdot \text{Risk}_{ij}^{\text{link}} \cdot \text{pop}_{ij}^{\text{link}} \\
F_3 & = \sum_{i} \sum_{d} R_i^d \cdot \text{Risk}_{i}^{\text{node}} \cdot \text{pop}_{i}^{\text{node}} \\
\sum_{j} x_{ij} & = 1 \hspace{0.5cm} \forall j \\
\sum_{i} x_{in} & = 1 \hspace{0.5cm} \forall i \\
\sum_{i} x_{ij} - \sum_{i} x_{ji} & = 0 \hspace{0.5cm}, \text{where} \hspace{0.5cm} j = 2, 3, \ldots, n-1 \\
a_i & = \text{start time} \\
a_j - \sum_{i} t_{ij} & = 0 \hspace{0.5cm}, \text{where} \hspace{0.5cm} \forall i, j = 2, 3, \ldots, n \\
q \cdot (1 - x_{ij}) + h_{ij} - w_i - c_{ij} & \geq 0 \hspace{0.5cm}, \forall (i, j) \in A \\
q \cdot x_{ij} - h_{ij} & \geq 0 \\
w_i + c_{ij} - h_{ij} & \geq 0 \hspace{0.5cm}, \forall (i, j) \in A \\
a_i + \sum_{d} R_i^d - \sum_{i} t \cdot u_{ij}^d - b_{ij} & = 0 \hspace{0.5cm}, \forall i, j \\
q \cdot x_{ij} + b_{ij} & \leq q \\
\sum_{i} u_{ij}^l - x_{ij} & = 0 \\
c_{ij} & = \sum_{i} v_{ij}^l \cdot u_{ij}^l = 0 \hspace{0.5cm}, \forall i, j \\
w_i & = 0 \\
w_j - h_{ij} & = 0 \hspace{0.5cm}, \text{where} \hspace{0.5cm} \forall i, j = 2, 3, \ldots, n \\
q \cdot (1 - x_{ij}) + h_{ij} - w_i - c_{ij} & \geq 0 \hspace{0.5cm}, \forall (i, j) \in A \\
q \cdot x_{ij} - h_{ij} & \geq 0
\end{align*}
The objective function (3.9) states that the weighted combination of the arrival time at the destination node n, population at risk along the route, and population at risk near the rest nodes should be minimum. The first three constraints, (3.10), (3.11) and (3.12) show how these components of the objective function are calculated respectively.

Constraints (3.13), (3.14) and (3.15) ensure that a path from origin node 1 to destination node n exists. In this model, each node and link has different information associated with it. These include: arrival time, cumulative work hour, rest hours, etc. There are constraints that calculate these attributes for each node and link so that all requirements are satisfied. The
relations among attributes, nodes, and links are shown in the model presented here mathematically.

Constraints (3.16) to (3.20) show the relation of arrival time between nodes and links. In conventional shortest path problems, the time when vehicles enter the link does not affect the link travel time. However, this study proposes a time varying network. The schedule when the vehicle begins its journey in each day affects the travel times of the links. The vehicle is allowed to leave the origin node and the rest nodes at any time to avoid congestion. Constraints (3.16) to (3.20) imply this special nature.

Constraint (3.17) is the arrival time at each node. Constraints (3.18), (3.19) and (3.20) compute the cumulative travel time of link (i, j). The travel time is zero if link (i, j) is not selected in the path. Based on these constraints, only the nodes that are on the path will have positive arrival times. Arrival times of other nodes that are not on the path are zero. The travel times of the links are time dependent. Constraints (3.21) to (3.24) calculate the link travel times. These constraints also ensure that the travel time of link (i, j) is equal to zero if link (i, j) is not in the path.

Constraints (3.25) to (3.29) calculate the absolute work hours without rest time. Since on each day the driver can’t drive more than k hours, when the cumulative work hours to reach a node exceeds the work limit, this node must be visited on the next day, and the driver must rest at the prior node. Constraints (3.30) and (3.31) ensure that $y_i^d$ equals one only when the node is traveled after the $d^{th}$ day. Based on $y_i^d$, $z_i^d$ can be obtained by constraint (3.32). Every node on the path will be assigned to one and only one day. Constraints (3.33) assign the arrival time to each node on the daily path, and all other nodes that are not on the daily path will be assigned a large negative number. Constraints (3.34) and (3.35) ensure the $L^d$ is the latest arrival time of each day.

Constraints (3.36) to (3.41) ensure that there is only one rest node in each day, and the rest node must be the last visited node in that day. Constraints (3.42) and (3.43) imply that the drivers must rest at least ten hours at the selected rest node.

Although the sub-objectives have different units, this does not pose a problem. We can define suitable weights to convert them into the same unit, such as dollars or fatalities per mile. We did not focus on the determination of these weights in this research. The weights are defined in this study arbitrarily to show the relative importance of each individual objective. Determination of proper weights is left for future research.

4. OPTIMIZATION MODEL TESTING

A Hypothetical Network, which consists of 17 nodes and 31 arcs, is constructed to illustrate the applicability of this multi-objective model. The network configuration is shown in Figure 2. Node 1 represents the hazardous material generation site and Nodes 17 represents the hazardous material destination site. The hazardous material carrier is scheduled to leave the generation site within the assigned time window and travel to the destination site on the planned route. The vehicle can depart from the start node after 7:00 AM. The maximum workload for each day is 12 hours after which the driver must rest for 10 hours at an assigned rest node. Congested times are from 7:00 to 10:00 and 15:00 to 18:00. It should be noted that only weekdays shipments are considered in this thesis. Traffic patterns on the weekends might be different. The rest of the day is considered as normal time. If a carrier enters a major highway during the congested time, it spends more time to pass the link than normal time.
This hypothetical problem is solved by CPLEX using different weight settings, and the results are shown in Table 1. In the first three cases, only one of the factors was taken into account. Case 1 shows the least risky route and its corresponding departure schedule, case 2 shows the least risky rest node and case 3 shows the shortest path and its corresponding departure schedule. These results are used to examine the formulation logic. For example, case 1 shows that if the link risk is the only concern, it takes 37 hours to arrive at the destination. This is 4 hours more than the result of case 3 (33 hours) that is the shortest path. Meanwhile, if the minimum cost is to be achieved, the tradeoff is a much higher rest node risk. Comparing cases 2 and 3, the rest nodes are nodes 12 and 13 respectively, and the node risk of node 13 is 9.55 times higher than the node risk of node 12.

Another special characteristic that can be noted in cases 4 and 5 (Table 2) is that when the node risk is considered in the formulation, a safer node is preferred as rest node. In general, the solutions tend to avoid the arterial traffic at peak hours and high-risk links and nodes as predicted. As the risk weight increases, the rest node is located in a remote node with lower population and risk rate.

Although the computational times are prohibitive while we increase the size of the network, the mathematical formulation generated the optimal solutions that are needed. The mathematical formulation is successful and provides optimal solutions that can be compared with heuristic solutions.

5. HEURISTIC APPROACH

Since the mathematical formulation that is solved by CPLEX cannot solve the problem within a reasonable time, a heuristic is proposed in this thesis to yield a good solution efficiently.

5.1. Proposed Heuristic

The basic idea of this heuristic can be described as follows. A time variable time period is selected so that the drivers must rest at a node during the period before the driver reaches the workload limitation. Those nodes at which the driver can arrive during the time period are called frontier nodes. The hazardous material carrier leaves the start node, the origin in the first day, during scheduled departure time. Then the minimum cost path from start node to each frontier node is assigned by labeling procedure. All information is stored in the solution set. Each frontier node becomes a start node in the next day, and new sets of frontier nodes as well as minimum cost paths are generated and stored. This process is repeated till the destination node is involved in the frontier nodes set. The best solution including the path and departure schedule can be found by tracing the solution set from the destination to the origin. The steps of the algorithm are outlined below:

Step 1. [Initialization]
1.1 Set workload, required rest hours, link risk weight, node risk weight, travel time weight, maximum allowed delay in hours, arrival time period of frontier nodes, and start time.
1.2 Initialize OPEN_DAY and SOLUTION sets.
1.3 Let Delay = 0, REST_HOUR = 0, and Day = 1
1.4 Pick origin to be the start node.
Step 2. [Search frontier]
2.1 Let departure time = start time + Delay.
2.2 Call Dijkstra’s algorithm.
2.3 Store all the frontier nodes in OPEN_DAY and required information in SOLUTION.
2.4 If Delay is less than or equal to maximum allowed delay hours, let Delay = Delay + 1 and go to step [2.1].

Step 3. [Expand]
3.1 If OPEN_DAY is not empty, select a new start node from OPEN_DAY and remove it from OPEN_DAY. Let REST_HOUR equal to the required rest hours, DAY = DAY + 1 and reset Delay = 0. Go to step 2.
3.2 If the OPEN_DAY is empty, go to [step 4].

Step 4. [Terminate]
4.1 If destination is included in SOLUTION, use SOLUTION set to find the best route and schedule.
4.2 Otherwise, go to step 3.

5.2. Performance Analysis

The comparison between mathematical results and heuristic results will be shown in this section and it will show that the heuristic functions efficiently and yields good solutions. The proposed heuristic was applied to the network shown in previous section. In addition, the heuristic was coded in C++ language and the mathematical formulation was solved by CPLEX. The platform was a Pentium 4 processor. The comparison is shown in Tables 3.

We focus on each special characteristic. Link risk, node risk, and cost risk are set as a main factor separately, while the other factors are ignored in this set. Table 3 shows that both the heuristic and CPLEX have the same objective value in all three cases. However, there are three interesting observations. First, in the first case the heuristic chose node 8 as a rest node but CPLEX chose node 12. This is not a big surprise, because in this case the focus was on link risk only. Therefore, the rest node would not affect the objective value but the heuristic and CPLEX did have the same route. The same situation happened in the third case, too. Second, an opposite situation happened in the second case. The heuristic and CPLEX suggested the same rest node and different routes since the second case focused on searching the minimum risk rest node. Third, the heuristic spent significantly less computation time than CPLEX. These first step tests show that the heuristic succeeded in these extreme scenarios. Nevertheless, further examinations were conducted.

We chose many different combinations of the weights to test the heuristic. In these cases, the link risk weight decreased from 0.9 to 0.6 while the node risk and travel time weight were increased.

It is quite clear that both the proposed heuristic and CPLEX have the same solution. In fact, the optimal routes have the same path in these cases. In some other cases, they might have the same objective value but different routes. However, the heuristic is still more efficient in solving this problem.

Although the heuristic generates the optimal solutions for all the cases presented above, one should note that it is not guaranteed that the heuristic generates the optimal solution always.
The arrival time period at the frontier node set greatly affects the performance of this heuristic. The arrival time period is a strategy used in the heuristic to generate the frontier node set and serves this sole purpose only. This strategy is not used in the mathematical formulation. Defining a time period width a priori enables the heuristic to search only part of the solution space and hence reduces the computation time. The mathematical formulation searches the whole solution space for the optimal solution. As mentioned in the previous section, the frontier nodes are the new origins on the next day. The arrival time period determines how many nodes will be selected as the origin nodes on the next day. If we choose a smaller time period, the computation time will decrease because the solution space that will be searched is reduced. Nevertheless, the optimal rest node may be excluded from the frontier set. Thus this heuristic will miss the optimal route and schedule.

Cases 12 to 15 show how time period affects the solutions. We use the same network configurations. Link risk weight, node risk weight and travel time weight are 0.5, 0 and 0.5 respectively. But we change the time period from 2 to 5 hours. In Case 12, the time period is 2 hours. This means only the nodes in which the trucks can arrive within 2 hours before the drivers’ daily workload is reached will be included in the frontier set. The time periods in cases 12, 13, 14 and 15 are 2, 3, 4 and 5 respectively. The comparison is shown in Table 4. The first column of Table 4 is the optimal solution from CPLEX. The last row of the table shows the gap between the optimal solution and the heuristic solution in each case. Table 4 compares the results from the heuristic and CPLEX. Case 12, which has the smallest time period, has a 15.17% gap from the optimal solution. Case 13 has a 13.29% gap from the optimal solution. The reason for this is that node 8 is not included in the frontier set. Therefore, the heuristic takes node 10 as a rest node not node 8. These biased results are not optimal routes and schedules. After we increase the time period to 4 and 5 hours, node 8 is included in the frontier set and the heuristic can generate the optimal solution as shown in cases 14 and 15. In summary, the solutions that we get from the heuristic highly depend on the time period setting. We recommend setting a larger time period that is at least 40% of the daily workload. By setting a larger time period, we have a better chance to get good solutions.

5.3. Implementation on an Expanded Network

An expanded network which contains 26 nodes is used to test the proposed heuristic’s efficiency and effectiveness. The expanded network is shown in Figure 3. In this expanded network, the origin node is node 1 and the destination node is node 26. The heuristic approach generates the solutions very efficiently and can be applied to a more sophisticated network. The results of the application of the method in several tests are shown in Table 5. These cases differ in the weight associated with different objectives.

We applied this heuristic to test problem developed on a network with 40 nodes. The results are shown in Table 6. The heuristic still generates the solutions very efficiently. We tested 10 different weight settings in Cases 11 through 20. If we focus on cost only, the travel time is the shortest among all cases. The other cases also have similar patterns as previous tested networks.

In this section, we presented a heuristic to solve this problem. The results from heuristic are compared with mathematical solutions in a small network and the heuristic results reproduced the optimal solutions in the test problem. The heuristic is also tested in a larger
5. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

In this study, we proposed to account for the driver’s workload and travel time variations in the network. These new features make this problem more realistic; however, more complex than before. The formulation provides for the route and location of rest points for the drivers simultaneously but it is very complicated.

A heuristic is developed to solve large scale problems. This heuristic generates a frontier node set every day, and the driver rests in one of these frontier nodes and then continues the journey on the next day until the destination is reached. Compared with CPLEX solutions, the application of the proposed heuristic in the test problems produced optimal solutions in a small network within an extremely short time. When this heuristic was applied to a larger scale problem, the computational time was never more than one second. This indicates that the proposed heuristic is an effective and efficient method for solving this problem. The fleet manager who deals with hazardous material shipments in a chemical or nuclear plant can use this research to design the routes for the shipments. The research described in this study can also be a useful foundation for further research.

The rest nodes and workload limitation have been shown as cardinal components in hazmat transportation. We need to define a more adequate equation to estimate the potential risk to the neighborhood when a hazmat carrier rests at a node or travels on the road. Emergency response capabilities of counties or cities through which the hazardous material is shipped are also important factors. Considering the location of emergency response facilities is left for future studies. Different kind of hazardous material, climate, and environment should be distinguished. Besides, the driver’s attention will decrease after he drives a long time. Therefore, it seems that the risk equation is also time dependent. This requires further investigation.

The routing problem in this study was a one origin and one destination problem. Sometimes, the carrier could be asked to pick up hazardous material from different locations and deliver to different destinations in a specified time windows. If we have multiple shipments from the same origin to the same destination, we may not want to use the same route for every shipment in order to spread out the risk. Therefore, we might want to split the shipments into different routes in this situation. Selection of the routes and determination of the number of trucks to use on each route in these circumstances are also interesting challenges.

Finally, link capacity was ignored in this study. If we have several carriers who are shipping hazardous materials from different origins to different destinations, or if we have a fleet of vehicles that use the same links, the interactions between the routing decisions is an important area for investigation. These present attractive areas for future research. This thesis can be considered a stepping-stone for development of more sophisticated models that can solve problems in which we have to deal with multiple shipments for one origin to one destination or multiple shipments from multiple origins to multiple destinations.
REFERENCES


List of Tables and Figures

Table 1 Routes and Schedules for cases 1 to 3 by CPLEX
Table 2 Routes and Schedules for cases 4 and 5 by CPLEX
Table 3 Comparisons between Heuristic and CPLEX Solutions (Cases 1 – 11)
Table 4 Comparisons between Heuristic and CPLEX Solutions (Cases 12 – 15)
Table 5 Solutions from 10 Cases using Heuristic Approach
Table 6 Solutions from Cases 11 through 20

Figure 1 Problem Description Flowchart
Figure 2 Hypothetical Network
Figure 3 Expanded Network
### Table 1
Routes and Schedules for cases 1 to 3 by CPLEX

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Objective Value</th>
<th>Case 2</th>
<th>Objective Value</th>
<th>Case 3</th>
<th>Objective Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link Risk Weight</td>
<td>Node Risk Weight</td>
<td>Cost Weight</td>
<td>Link Risk Weight</td>
<td>Node Risk Weight</td>
<td>Cost Weight</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Node</td>
<td>Arrival Time</td>
<td>Departure Time</td>
<td>Rest Hour</td>
<td>Node</td>
<td>Arrival Time</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>12</td>
<td>0</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>15</td>
<td>0</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>19</td>
<td>29</td>
<td>10</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>15</td>
<td>34</td>
<td>34</td>
<td>0</td>
<td>13</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>37</td>
<td>37</td>
<td>0</td>
<td>14</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>17</td>
<td>40</td>
</tr>
</tbody>
</table>
Table 2  
Routes and Schedules for cases 4 and 5 by CPLEX

<table>
<thead>
<tr>
<th>Case 4</th>
<th>Objective Value</th>
<th>15.2938</th>
<th>Case 5</th>
<th>Objective Value</th>
<th>26.548</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link Risk Weight</td>
<td>Node Risk Weight</td>
<td>Cost Weight</td>
<td>Link Risk Weight</td>
<td>Node Risk Weight</td>
<td>Cost Weight</td>
</tr>
<tr>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Arrival Time</th>
<th>Departure Time</th>
<th>Rest Hour</th>
<th>Node</th>
<th>Arrival Time</th>
<th>Departure Time</th>
<th>Rest Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>5</td>
<td>9</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>12</td>
<td>0</td>
<td>4</td>
<td>12</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>15</td>
<td>0</td>
<td>8</td>
<td>15</td>
<td>26</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>19</td>
<td>29</td>
<td>10</td>
<td>12</td>
<td>30</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>34</td>
<td>34</td>
<td>0</td>
<td>15</td>
<td>35</td>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>37</td>
<td>-</td>
<td>-</td>
<td>17</td>
<td>37</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 3
Comparisons between Heuristic and CPLEX Solutions (Cases 1 – 11)

<table>
<thead>
<tr>
<th>Case</th>
<th>Heuristic</th>
<th>Link Risk Weight</th>
<th>Node Risk Weight</th>
<th>Travel Time Weight</th>
<th>Route</th>
<th>Rest Node</th>
<th>Rest Hour</th>
<th>Objective Value</th>
<th>Comput. Time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Heuristic</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1,5,4,8,12,15,17</td>
<td>8</td>
<td>10</td>
<td>16.096</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.096</td>
<td>14.96</td>
</tr>
<tr>
<td>2</td>
<td>Heuristic</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1,5,9,12,15,17</td>
<td>12</td>
<td>10</td>
<td>6.454</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,9,12,15,17</td>
<td>12</td>
<td>10</td>
<td>6.454</td>
<td>3368</td>
</tr>
<tr>
<td>3</td>
<td>Heuristic</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1,2,6,10,13,15,17</td>
<td>10</td>
<td>10</td>
<td>33</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,2,6,10,13,15,17</td>
<td>13</td>
<td>10</td>
<td>33</td>
<td>2098.68</td>
</tr>
<tr>
<td>4</td>
<td>Heuristic</td>
<td>0.9</td>
<td>0.1</td>
<td>0</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>15.1318</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>15.1318</td>
<td>5.56</td>
</tr>
<tr>
<td>5</td>
<td>Heuristic</td>
<td>0.8</td>
<td>0.1</td>
<td>0.1</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>17.2222</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>17.2222</td>
<td>9.06</td>
</tr>
<tr>
<td>6</td>
<td>Heuristic</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.258</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.258</td>
<td>11.83</td>
</tr>
<tr>
<td>7</td>
<td>Heuristic</td>
<td>0.7</td>
<td>0.1</td>
<td>0.2</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>19.3126</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>19.3126</td>
<td>12.77</td>
</tr>
<tr>
<td>8</td>
<td>Heuristic</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>15.2938</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>15.2938</td>
<td>17.9</td>
</tr>
<tr>
<td>9</td>
<td>Heuristic</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.656</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.656</td>
<td>0%</td>
</tr>
<tr>
<td>10</td>
<td>Heuristic</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.102</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>16.102</td>
<td>0%</td>
</tr>
<tr>
<td>11</td>
<td>Heuristic</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>1,2,6,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>20.312</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPLEX</td>
<td></td>
<td></td>
<td></td>
<td>1,2,6,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>20.312</td>
<td>0%</td>
</tr>
</tbody>
</table>
Table 4
Comparisons between Heuristic and CPLEX Solutions (Cases 12 – 15)

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Period</th>
<th>Route</th>
<th>Rest Node</th>
<th>Rest Hour</th>
<th>Objective Value</th>
<th>GAP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPLEX</td>
<td>1,5,4,8,12,15,17</td>
<td>8</td>
<td>11</td>
<td>26.548</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Heuristic</td>
<td>1,5,6,10,14,16,17</td>
<td>10</td>
<td>10</td>
<td>30.576</td>
<td>15.17%</td>
</tr>
<tr>
<td>13</td>
<td>Heuristic</td>
<td>1,5,6,10,14,16,17</td>
<td>10</td>
<td>10</td>
<td>30.076</td>
<td>13.29%</td>
</tr>
<tr>
<td>14</td>
<td>Heuristic</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>26.548</td>
<td>0%</td>
</tr>
<tr>
<td>15</td>
<td>Heuristic</td>
<td>1,5,4,8,12,15,17</td>
<td>12</td>
<td>10</td>
<td>26.548</td>
<td>0%</td>
</tr>
</tbody>
</table>
Table 5
Solutions from 10 Cases Using Heuristic Approach

<table>
<thead>
<tr>
<th>Case</th>
<th>Link Risk Weight</th>
<th>Node Risk Weight</th>
<th>Cost Weight</th>
<th>Route</th>
<th>Rest Node</th>
<th>Rest Hour</th>
<th>Objective Value</th>
<th>Arrival Time</th>
<th>Comput. Time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>14.5607</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,17,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1,3,5,8,11,12,15</td>
<td>11</td>
<td>10</td>
<td>5.3097</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19,23,22,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1,3,4,6,9,12,11</td>
<td>12</td>
<td>10</td>
<td>53</td>
<td>53</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14,18,22,25,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>0</td>
<td>0.1</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>19.3046</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,17,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>0</td>
<td>0.2</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>24.0485</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,17,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>32.0081</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,17,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>54.1602</td>
<td>53</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18,22,25,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>1,3,4,6,9,12,11,</td>
<td>11</td>
<td>10</td>
<td>29.9045</td>
<td>55</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12,15,19,23,25,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>38.2803</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,17,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1,3,5,8,11,14</td>
<td>11</td>
<td>10</td>
<td>35.7331</td>
<td>62</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13,17,21,24,26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Table 6
Solutions from Cases 11 through 20

<table>
<thead>
<tr>
<th>Case</th>
<th>Link Risk Weight</th>
<th>Node Risk Weight</th>
<th>Cost Weight</th>
<th>Route</th>
<th>Rest Node</th>
<th>Rest Time Hour</th>
<th>Objective Value</th>
<th>Arrival Time</th>
<th>Computation Time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>21.5471</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1,27,3,26,29,34,11,12,32,15,19,20,23,22,24,40</td>
<td>11</td>
<td>10</td>
<td>18.8547</td>
<td>67</td>
<td>&lt;1</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1,27,3,28,5,8,12,32,14,38,18,21,24,40</td>
<td>5</td>
<td>10</td>
<td>59</td>
<td>59</td>
<td>&lt;1</td>
</tr>
<tr>
<td>14</td>
<td>0.9</td>
<td>0</td>
<td>0.1</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>25.7924</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>15</td>
<td>0.8</td>
<td>0</td>
<td>0.2</td>
<td>1,27,3,26,29,34,11,33,14,38,18,21,24,40</td>
<td>11</td>
<td>15</td>
<td>41.6146</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>16</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>24.2841</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>17</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>55.0066</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>40.2597</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>19</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>42.7736</td>
<td>64</td>
<td>&lt;1</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1,27,3,26,29,34,11,33,14,38,19,23,25,40</td>
<td>11</td>
<td>10</td>
<td>19.0333</td>
<td>64</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>
Figure 1
Problem Description Flowchart
Figure 2
Hypothetical Network
Figure 3
Expanded Network