A Method for Capacity Analysis Based on Theoretical and Practical Considerations

Submitting Date: July 31, 2002

Word Count: 5660

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Submitted to the TRB 82nd Annual Meeting and TRR

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ABSTRACT

The current unprecedented development of freeway systems in China has been one of the largest infrastructure investments in recent Chinese history. Capacity analysis plays a key role in planning, construction, and classification of the Chinese freeway systems. Incorrect capacity analysis could lead to either an overbuilt freeway system wasting valuable resources or an under-built freeway system that cannot meet the demand resulting from the rapid economic development. Over the past decade, many studies have been conducted regarding the freeway capacity analysis and performance evaluation in China based on the procedures and methodologies established in the developed countries such as Japan and the United States. However, because of the significant differences in cultural background, highway systems, as well as the economic development stages, the procedures from the developed countries cannot be directly applied to the Chinese freeway capacity analysis. Additionally, it has also been recognized that the current produces for highway capacity analysis from the developed countries still have their limitations in both theoretical basis and practical applications.

This paper presents a method for freeway capacity analysis by linking the traffic flow model with the highway safety requirements. The capacity derived from this method is theoretically sound and practically feasible.
REVIEW

A basic freeway segment is defined as a section between two ramps (either entrance or exit) or interchanges that is not affected by weaving and diverging maneuvers. The traffic flows continuously on a basic freeway segment. The current freeway capacity analysis (1) in China is mainly based on the Highway Capacity Manual from the United States (2) or the procedures established in Japan. In both cases the freeway capacity is generally determined by the following three steps:

Theoretical Capacity

Under ideal traffic and environmental conditions, the basic capacity or theoretical capacity is derived from the concept that the flow rate equals time divided by the time headway; and time headway equals distance divided by speed as shown below (in metric system):

\[
q = \frac{3600}{t}
\]

\[
t = \frac{S}{V}
\]

\[
S = V\delta + \frac{V^2}{2g(f \pm G)} + l_s + l_v
\]

where  
- \( q \) = flow rate (pc/h, number of passenger cars per hour)  
- \( t \) = time headway (s)  
- \( S \) = safe stopping distance  
- \( V \) = speed (m/s)  
- \( \delta \) = perception-reaction time (s)  
- \( f \) = coefficient of friction (between tire and pavement)  
- \( l_s \) = safety margin between vehicles (m)  
- \( l_v \) = average vehicle length (m)  
- \( g \) = gravity constant  
- \( G \) = slope in percentage

with \( G=0 \), \( g=9.8m/s^2 \), and \( V \) in kilometers per hour:

\[
q = \frac{1000V}{\frac{V}{3.6}\delta + \frac{V^2}{25\phi} + l_s + l_v}
\]

Since flow rate is a function of the vehicle speed, the maximum flow rate or the capacity can be achieved by:

\[
\frac{dq}{dV} = 0
\]
from which

\[ V_m = \sqrt{254\phi(l_s + l_v)} \]

\[ C_B = \frac{3600}{\delta + 7.2 \sqrt{\frac{l_s + l_v}{254\phi}}} \]  \hspace{1cm} (2)

where \( V_m = \text{speed at the capacity} \)
\( C_B = \text{capacity}. \)

The capacity obtained from Equation 2 is generally between 1100 and 1600 vehicles per hour per lane, which is far less than the observed maximum traffic flow on freeways. Therefore, the actual capacity is not directly determined by this theoretical equation, but rather from the actual observations in Japan and the United States.

**Prevailing Capacities**
Because the freeway conditions, such as the design and traffic composition, vary greatly, the prevailing capacity for a freeway segment is determined by multiplying the ideal capacity by the adjustment factors as recommended by the U.S. Highway Capacity Manual (HCM). A basic capacity is determined based on the repeatedly observed maximum traffic flow under ideal traffic and environmental conditions. For any segment of freeways operating under non-ideal conditions, its capacity will always be smaller than the basic capacity. Thus,

\[ C_p = C_B \times \sum_j f_j \]  \hspace{1cm} (3)

where \( C_p = \text{prevailing capacity} \)
\( f_j = \text{adjustment factor for the condition } i. \)

**Level of Service and Design Service Rate**
The level of service is used to classify the operating performance of a highway facility. A design service flow rate under a given level of service on freeway can be computed by:

\[ C_{LOS=I} = C_B \times \left( \frac{v}{C_B} \right)_{LOS=I} \]  \hspace{1cm} (4)

where \( C_B = \text{capacity under ideal condition} \)
\( v = \text{volume}. \)

**PROBLEMS WITH EXISTING METHODS**
Based on the above reviews, the shortcomings of both theoretical capacity and prevailing capacity, as well as the method of determining safe freeway operation in the current Chinese highway system, are summarized in the following paragraphs:
1. The theoretical capacity does not reflect reality; thus, it cannot be used in the real-world applications. Although some modifications have been made over the years, it still lacks the analytical ability to explain the internal mechanism of traffic flow. The failure of a theoretical capacity model had resulted in the dependence on the field observations for capacity. Being short of theoretical guidance, the field observations always have inherent uncertainties.

2. It does not analytically explain why the actual capacity of a segment of freeway can be determined by applying the adjustment factors to an ideal capacity. Due to the lack of a clear mechanism, it is impossible to compare and evaluate the adjustment factors used in Japan and the United States, and thereby make necessary modifications to suit the conditions of the Chinese freeway systems.

3. The well-developed modern traffic flow theory has revealed an internal relationship among flow rate, density, and speed. Flow rate is generally a function of density or speed with a maximum value on its convex shape of curve. Theoretically, this maximum point represents the capacity. To use this value as the capacity, it is essential to know whether the maximum flow rate reasonably meets the highway safety requirement. Since the determination of the maximum flow rate based on the flow-speed or flow-density curves is purely mathematical and totally independent of Equation 3 for actual capacity, there are often significant differences between the two values even though they represent same concept.

4. To promote a safe highway operation, it is required to have a safe stopping distance for the Chinese freeway operating systems. The corresponding density can be computed for any given speed and distance headway as shown in Table 1. To meet this safety requirement, freeways must operate under high level of service that is neither reasonable nor productive.

The above-discussed problems have been causing some confusion in the development of a method for the capacity analysis that is both applicable and theoretically sound for the Chinese freeway systems. This paper presents a methodology for freeway capacity analysis that provides theoretical explanations and accommodates the safety requirements as well.

**ANALYSIS OF DISTANCE HEADWAY**

The distance headway and speed directly determine the capacity. To understand the concept of capacity, it is essential to understand the relationship between speed and distance headway. The feasible range for speed and distance headway can be illustrated by the speed-distance diagram shown in Figure 1, where \( l \) represents the minimum distance headway, i.e., a vehicle’s physical dimension plus the minimum gap between two vehicles, and \( V_f \) the maximum speed or the free-flow speed. Generally, the distance headway increases as the speed increases. The points on the \( h \)-axis represent the stop
condition with different densities, while the points on the horizontal line at distance $l$ from the $V$-axis represent the racecar situation. The vertical line at $V_f$ represents the free-flow condition. The hatched area bounded by these three lines represents a feasible range for speed and distance headway.

**Safe Distance Headway**

Although any point within the feasible range represents a possible vehicle-following condition, each point indicates a difference level of safety. To avoid collision, the *general distance headway* is determined by travel speed and deceleration rate from Equation 5 (3).

$$h_g = l + V\delta + \frac{V^2}{2d_2} - \frac{V^2}{2d_1}$$  \hspace{1cm} (5)

*where* $l =$ *vehicle length plus minimum gap between two successive vehicles* (m)  
$V =$ *speed* (m/s)  
$d_2 =$ *following vehicle’s deceleration rate* (m/s²)  
$d_1 =$ *leading vehicle’s deceleration rate* (m/s²).

Safe distance headway varies with the speed and deceleration rates of the leading and following vehicles. The difference between two vehicles’ deceleration rates results in different levels of safety. Generally, the leading vehicle brakes harder than the following vehicle ($d_1 \geq d_2$) because the following vehicle can detect the situation by the brake lights of the leading vehicle. Under the extreme case, the leading vehicle stops instantly ($d_1 =$ infinite); there is then the *sufficient distance headway*:

$$h_s = l + V\delta + \frac{V^2}{2d_2}$$  \hspace{1cm} (6)

If deceleration rates of both vehicles are the same, only the first two terms in Equation 6 remain. This yields the *minimum distance headway* as shown in Equation 7.

$$h_m = l + V\delta$$  \hspace{1cm} (7)

The distance headways defined above provide the safety requirement under three operating situations. The degree of safety increases as the required distance headway increases from the minimum distance headway in Equation 7, to the general distance headway in Equation 5, and then to the sufficient distance headway in Equation 6. Although the minimum distance headway provides a safe operation, it is hard to meet this requirement for motorists in the real world situation. The observed distance headway varies depending on the highway geometric design, traffic condition, and characteristics of the individual motorist. When motorists’ desired safe distance headway matches the required safe operation condition, the actual distance headway is close to the required safe distance headway. Thus, from the traffic management’s point of view, it is important...
to establish the proper traffic control devices and regulations to reduce the gap between
the desired and actual safe distance headways.

**Desired Distance Headway**

To drive efficiently and comfortably, motorists have their own desired distance headway
\( (h_d) \) based on the perceived highway environment and traffic condition. Under a stable
traffic flow, the desired distance headway can be easily achieved. When there is a
difference between the obtainable distance headway and desired distance headway,
motorists have to adjust their speed to adapt the situation. Sometimes, the adjustment
could lead to unstable traffic flow.

In general, the selection of the headway by a motorist is affected by three factors: safety,
efficiency and comfort. Safety and comfort require longer distance headway, while
efficiency calls for shorter distance headway between vehicles. The risky distance
headway refers to the situation where the actual (obtainable) distance is smaller than the
minimum distance headway; the conservative distance headway refers to the situation
where the actual (obtainable) distance is longer than the minimum distance headway; and
the ideal flow condition occurs when the obtainable distance headway equals the
minimum distance headway.

Selecting the desired distance headway is a fuzzy process that involves many factors,
which are hard to model quantitatively. The most influential factor is speed. The general
relationship between speed and distance headway can be observed and studied with
statistical analysis. The linear relationship between speed and density described by the
famous Greenshields (4) model can be used to derive the relationship between speed and
desired distance headway as follows:

\[
k = k_j (1 - \frac{V}{V_f})
\]

(8)

where \( k_j = \text{jam density (number of vehicles per meter, } \frac{v}{m}) \)
\( V_f = \text{free-flow speed (m/s)}. \)

With \( k = \frac{1}{h_e} \) and \( k_j = \frac{1}{l} \), the desired distance headway becomes:

\[
h_e = l \times \frac{V_f}{V_f - v}
\]

(9)

Figure 2 depicts the distance headway as a function of speed for four different models. As
expected, as the speed increases, the distance headway increases monotonously. At any
given speed, there is the relationship: \( h_r > h_s > h_m \). Within the feasible range of speed
and distance as displayed in Figure 1, the points below the curve of the minimum
distance headway represent the risky operation, while the points above this curve indicate
the conservative operation as discussed previously.

The curve representing the desired distance headway defined by Equation 9 approaches
infinity when the speed is close to the free-flow speed, and it intersects the minimum
distance headway curve at speed \( V_p \) as shown in Figure 2. When \( v < v_p \), the desired
distance headway curve is below the minimum distance headway; when \( v > v_p \), the
desired curve is above the curve of minimum distance headway. In other words, as speed
increases, the operating condition changes from the risky to the conservative type. This
change is understandable since, at the low speed, the efficiency generally overweighs
safety; and when speed is high, motorists concern more about safety and comfort. It is
particularly true on high-speed freeways where the desired distance headway is generally
far above the minimum distance headway for the majority of motorists.

Letting Equation 7 equal Equation 9, \( V_p \) can be calculated by:

\[
V_p = V_f - \frac{l}{\delta} \tag{10}
\]

The safe distance headways defined by Equations 5, 6 and 7 are independent of the free-
flow speed, while the desired distance headway is a function of the free-flow speed. As
the free-flow speed decrease, the curve representing the desired distance headway moves
upward. Mathematically when \( V_f \leq \frac{l}{\delta} \), \( V_p \leq 0 \), the desired distance curve is entirely
above the curve of the minimum distance headway. Higher free-flow speed results in a
larger portion of the desired distance headway positioned above the curve of the
minimum distance headway.

**DETERMINATION OF CAPACITY**

Based on the fundamental relationship between flow rate, speed and density \( q = k \times V \)
and \( k = \frac{1}{h} \), the relationship between flow rate and speed for the four headway distance
models are shown in Figure 3. All four curves are within a triangle defined by the
horizontal axis of speed, vertical line at \( V = V_f \), and a straight line of \( q = \frac{V}{l} = V \times k_j \) that
represents the racetrack conditions with a fixed density or distance headway. Any point
on or within this triangle theoretically represents a feasible traffic flow state.

Based on its definition, the maximum flow rate at point A in Figure 3 should represent
the capacity that is the product of the free-flow speed and jam density \( (C = V_f \times k_j) \). In
reality, this value is, however, not attainable because of safety requirements. Different
level of safety requirements leads to a different maximum flow rate, as manifested in
Equation 11:
\[ q = \frac{v}{h_i} \]  
where \( i = g, s, m, \) and \( e. \)

The maximum flow rate associated with different levels of safety can be determined from the above equation and Equations 5, 6, and 7. When \( h \) equals \( h_e \) of the Greenshields model in Equation 9, the flow rate becomes:

\[ q = \frac{v}{l} (1 - \frac{v}{v_f}) \]  
\[ (12) \]

This equation can be used to determine the maximum flow rate for the desired distance headway that is the curve 4 in Figure 3. At any given speed, a lower flow rate yields a higher level of safety.

**Basic Capacity**

From the analysis of the safe distance headway, it is clear that the minimum distance headway can reasonably represent a safe driving condition. Thus, the flow rate model derived accordingly represents an ideal flow condition that gives the maximum flow rate meeting the minimum safety requirement. This maximum flow rate or capacity can be determined by Equation 13:

\[ q = \frac{1000V}{l + \frac{V\delta}{3.6}} \]  
\[ (13) \]

where \( q = \text{flow rate} \ (v/h) \)

\( l = \text{minimum gap between vehicles} \ (m). \)

Equation 12 is represented by a solid line in Figure 4 that monotonously increases as the speed increases and approaches a horizontal line represented by \( C' = \frac{3600}{\delta} \). With the free-flow speed \( V_f \) under the ideal driving condition, the corresponding flow rate, \( C_{fb} \), is then the basic capacity. Replacing \( V \) with \( V_f \) in Equation 13, the maximum flow rate under basic safety condition becomes:

\[ C = \frac{1000V_f}{l + \frac{V_f\delta}{3.6}} \]  
\[ (14) \]

Based on a recent survey conducted in China (5), the observed minimum distance headway is about 6.5 to 8 meters for standard passenger cars, and the drivers’ perception-reaction time is between 0.8 to 1.2 seconds. With a minimum headway of 8 meters, the
perception-reaction time of 1.2 seconds, and the free-flow speed of 120 km per hour, the basic capacity will be 2500 passenger cars per hour per lane with the minimum distance headway of 48 meters. These values are very close to the data collected in the United States during the past several years. The upper limit of capacity is 3000 passenger cars per hour based on 
\[ C_{\text{b, max}} = \frac{3600}{1.2}. \]

Note that in the discussion of the basic capacity, the ideal driving condition is emphasized. This emphasis is important since when all other parameters are the same, various safety requirements lead to different capacities. For instance, the capacity derived from the sufficient distant headway is certainly smaller than the capacity from the minimum distance headway, and the corresponding speed, \( v_m \), is not the maximum free-flow speed. Therefore, it is neither reasonable nor practical to discuss the capacity while ignoring the safety requirement that is analytically presented by Equations 5, 6, and 7. When a level of safety is properly selected, the capacity derived from the desired distance headway closely matches the repeatedly observed maximum flow rate. Otherwise, the analytical capacity differs from the observed maximum flow rate. The capacity derived from the sufficient distance headway with the current method for capacity analysis in China is too conservative to be close to reality, and thus, not reasonable.

**Practical Capacity**

Due to the variation of many factors, such as the roadway lateral clearance, speed limit, composition of the vehicles, it is rare to have a perfect driving condition defined by the ideal geometric design, traffic composition, and motorists’ behavior in the real world situation. How to determine the capacity under prevailing conditions is a key issue for traffic engineers.

Assuming that under a given prevailing condition the free-flow speed is \( V_f \), and the desired distance headway is from the Greenshields model, the relationship between speed and flow rate is a parabolic curve as shown in Figure 4. The maximum flow rate occurs at point \( m \) that is often defined as the capacity under the given condition. However, if this point is above the flow curve derived from by the minimum distance headway, this capacity represents a risky traffic state as discussed previously. It is neither safe nor practical to use this value as the capacity.

Therefore, the practical capacity is the maximum flow rate that meets the safety requirement under the prevailing condition defined by the geometric design, traffic composition, and motorists’ characteristics. According to this definition, the maximum flow rate indicated by point \( m \) in Figure 4 should not be considered as the capacity if it is above the flow-rate curve derived from the minimum distance headway. Since the intersection at \( P \) in Figure 4 represents the maximum flow rate meeting the minimum safety requirement, it should represent the capacity with the corresponding speed \( V_p \).

From Equation 10, \( V_p = V_f - \frac{3.6l}{\delta} \), Equation 13 becomes;

\[ C_p = \frac{3600}{\delta} \left(1 - \frac{3.6 \times l}{V_f \times \delta} \right) \]  

(15)
On the other hand, when point \( m \) is below the flow rate curve derived from the minimum distance headway, it represents the true capacity. Based on the characteristics of the linear Greenshields model, the capacity can be expressed as

\[
C_p = \frac{1}{2} k_j V_m = \frac{1000}{4 \times l} V_f = \frac{250 V_f}{l}, \text{ with jam density } k_j = \frac{1000}{l}.
\]

Note that the position of point \( m \) depends on the free-flow speed; as the free-flow speed decreases, the value of \( m \) decreases as shown in Figure 5. When the free-flow speed equals \( \frac{7.2 \times l}{\delta} \), the points of \( m \) and \( P \) meet, i.e., \( V_p = V_n \). This point represents the threshold that varies along a straight line with a zero-intercept and a slope \( \frac{k_j}{2} \) as shown in Figure 5. For example, if the minimum gap \( l \) is 8 meters and the perception-reaction time 1.2 seconds, the free flow-speed on this threshold line will be 48 kilometers per hour.

In summary, the practical capacity can be determined by the following equations:

\[
C_p = \frac{3600}{\delta} \left( 1 - \frac{3.6 \times l}{V_f \times \delta} \right) \quad \text{(16)}
\]

when \( V_f < \frac{7.2 \times l}{\delta} \), \( V_p = V_f - \frac{3.6 \times l}{\delta} \), \( k_p = \frac{3600}{V_f \times \delta} \), \( h_p = \frac{V_f \times \delta}{3.6} \), and

\[
C_p = \frac{250 \times V_f}{l} \quad \text{(17)}
\]

when \( V_f \leq \frac{7.2 \times l}{\delta} \), \( V_p = \frac{V_f}{2} \), \( k_p = \frac{500}{l} \), \( h_p = 2l \).

With the different free-flow speeds, the actual capacities and the corresponding speeds and densities are listed in Table 2.

From the results of the analysis in this paper, the capacity should be determined based on the minimum safety requirement. The capacity represents the maximum flow rate under the prevailing operating conditions. When the free-flow-speed is higher than the threshold value, the actual or true capacity is smaller than that represented by the vertex of the parabolic flow curve. The peak value of the analytic flow model only represents the capacity when a free-flow speed is smaller than the threshold value.
DISCUSSION AND CONCLUSIONS
The method for the capacity analysis presented in this paper clearly explains why applying the traffic flow model or the safe distance headway model alone cannot always yield an appropriate capacity. More interestingly, this method analytically validates the capacities used in the U.S. Highway Capacity Model, which are based on the repeatedly observed maximum flow rates. Although the HCM does not offer theoretical interpretation for these maximum values, it is clear that they satisfy safety requirements. Linking the traffic flow model with the distance headway models makes the method for capacity analysis not only theoretically sound but also practical.

The proposed capacity models expressed by Equations 16 and 17 reveal that the most influential elements in the determination of the capacity are the traffic flow model and free flow-speed. It is well known that the determination of both elements depends heavily on the highway geometric design, traffic condition, as well as the characteristics of motorists. In general, the traffic flow model reflects, to a great extent, motorists’ behaviors. Variations in motorists’ trip purpose, occupation, and safety expectation lead to different desired headways. From the safety perspective, the desired headway must be larger or at least equal to the safe distance headway, in which a smaller capacity may be resulted.

All three elements, namely, the highway design, traffic condition, and motorist’s characteristic affect the free flow-speed, and hence influence the capacity. The relationship among these three elements is rather complex and difficult to model quantitatively. Motorists’ behavior is the result of these three elements. A free flow-speed and capacity will be reduced under non-ideal traffic condition.

It is rather complicated to select a traffic flow model that gives the desired headway distance suitable to a prevailing condition. There are many developed flow models that can be used for capacity studies. The selection of a proper traffic flow model is beyond the scope of this paper. Although the capacity analysis in this paper is based on the Greenshields model, the logic can be easily generalized for other traffic flow models.

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**TABLE 1 The Required Distance Headway between Vehicles on Freeway**

<table>
<thead>
<tr>
<th>Speed (km/h) ≥</th>
<th>120</th>
<th>100</th>
<th>80</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Headway (m) ≥</td>
<td>200</td>
<td>150</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>Density (Vehicles/km) ≤</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>
TABLE 2 Capacity with Different Free Flow-Speed

<table>
<thead>
<tr>
<th>Free Flow-Speed (km/h)</th>
<th>120</th>
<th>100</th>
<th>80</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practical Capacity (cars/h/lane)</td>
<td>2400</td>
<td>2280</td>
<td>2100</td>
<td>1800</td>
</tr>
<tr>
<td>$V_p$ (km/h)</td>
<td>96</td>
<td>76</td>
<td>56</td>
<td>36</td>
</tr>
<tr>
<td>$K_p$ (cars/km/lane)</td>
<td>25</td>
<td>30</td>
<td>38</td>
<td>50</td>
</tr>
<tr>
<td>$h_p$ (m)</td>
<td>40</td>
<td>33</td>
<td>27</td>
<td>20</td>
</tr>
</tbody>
</table>
FIGURE 1 The Speed and Distance Diagram.
FIGURE 2 The Desired and Safe Distance Headway.
1. Minimum Distance Headway
2. General Distance Headway
3. Sufficient Distance Headway
4. Desired Distance Headway

\[ q = V \times k_j \]

FIGURE 3 The Relationship between Speed and Flow Rate.
FIGURE 4 The Basic and Actual Capacity.

$q = k_j V$

$b' = \text{basic capacity with } V_f$

$b = \text{basic capacity with } V'_{f}$
FIGURE 5  The Impact of $V_f$ on the Actual Capacity.

$q = \frac{1}{2} k_j V$

$q = \frac{1}{2} k_j V_m$